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of the United States Geological Survey

Chapter A5

**MEASUREMENT OF PEAK DISCHARGE  
AT DAMS BY INDIRECT METHOD**

By Harry Hulsing

Book 3

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## PREFACE

The series of manuals on techniques describes procedures for planning and executing specialized work in water-resources investigations. The material is grouped under major subject headings called books and further subdivided into sections and chapters; Section A of Book 3 is on surface water.

The unit of publication, the chapter, is limited to a narrow field of subject matter. This format permits flexibility in revision and publication as the need arises.

Provisional drafts of chapters are distributed to field offices of the U.S. Geological Survey for their use. These drafts are subject to revision because of experience in use or because of advancement in knowledge, techniques, or equipment. After the technique described in a chapter is sufficiently developed, the chapter is published and is for sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

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## SYMBOLS AND UNITS

<i>Symbol</i>	<i>Definition</i>	<i>Unit</i>		<i>Symbol</i>	<i>Definition</i>	<i>Unit</i>
<i>B</i>	Width of channel.	ft		$h_s$	Piezometric head measured from crest of sharp-crested weir used as reference weir for computing discharge coefficient for nappe-fitting weirs.	ft
<i>b</i>	Width of weir crest normal to flow.	ft		$h_t$	Vertical distance from the dam crest to the downstream water surface for flow affected by tail-water submergence.	ft
<i>C</i>	Coefficient of discharge.			$h_v$	Velocity head at a section.	ft
$C_i, C_o$	Coefficient of discharge for a weir for the cases $H = H_i$ , and for flow affected by tail-water submergence.			<i>k</i>	Adjustment factor; subscripts refer to specific items as: <i>c</i> for side contraction, <i>j</i> for piers and piles, <i>R</i> for radius, <i>s</i> for slope of downstream face of weir, <i>t</i> for submergence.	
$E_1, E_2$	Slope of upstream and downstream weir face, horizontal distance/vertical distance.			<i>L</i>	Length of broad-crested weir in direction of flow.	ft
<i>E</i>	Rise in lower nappe from spring point to its highest elevation.	ft		<i>P</i>	Height of weir above channel bed.	ft
<i>f</i>	Function of.			<i>R</i>	Radius of rounding, upstream edge, broad-crested weir.	ft
<i>g</i>	Gravitational constant (acceleration).	ft/sec <sup>2</sup>		<i>r</i>	Radius of side abutment rounding.	ft
<i>H</i>	Total head on a weir measured from high point on crest; $H = h + h_v$ .	ft		<i>V</i>	Mean velocity of flow in a section.	ft/sec
$H_i$	Index head, a value of <i>H</i> for which the coefficient of discharge is known.	ft		<i>x</i>	Horizontal coordinate of a point on the lower nappe surface of flow over a sharp-crested weir.	ft
$H_o$	Total head corresponding to the design head, $h_o$ , on nappe-shaped weirs; $H_o = h_o + h_v$ .	ft		<i>y</i>	Vertical coordinate of a point on the lower nappe surface of flow over a sharp-crested weir.	ft
<i>h</i>	Static or piezometric head above an arbitrary datum.	ft		<	Less than.	
$h_a$	Static head referred to high end of weir.	ft		=	Equal to or less than.	
$h_b$	Static head referred to low end of weir.	ft		>	Greater than.	
$h_o$	Nappe-fitting piezometric head referred to the high point on crest of dam $= h_s - E$ .	ft		≧	Equal to or greater than.	

# MEASUREMENT OF PEAK DISCHARGE AT DAMS BY INDIRECT METHODS

By Harry Hulsing

## Abstract

This chapter describes procedures for measuring peak discharges using dams, weirs, and embankments. Field and office procedures limited to this method are described. Discharge coefficients and formulas are given for three general classes of weirs—sharp-crested, broad-crested, and round-crested—and for highway embankments and weirs of unusual shape. The effects of submergence are defined for most forms.

## Introduction

A weir, dam, or embankment generally forms a control section at which the discharge is related to the upstream water-surface elevation. The peak discharge at the control section can usually be determined on the basis of a field survey of high-water marks and the geometry of the particular structure. These methods are derived from investigations of the discharge characteristics of weirs, dams, and embankments, as reported in the literature, and from laboratory studies made by the U.S. Geological Survey.

There are three fairly distinct general classes of weirs—sharp-crested, broad-crested, and round-crested. This immediately suggests the organization for this part of the report. However, many weirs do not neatly fit into these three classes, because of shape, physical condition of the channel, or the hydraulic conditions under which the weirs are operating. For example, a sharp-crested weir may act like a broad-crested weir because of fill in the approach channel, or a broad-crested weir may act like a sharp-crested weir because the head is sufficient to make the nappe spring clear. It is thus necessary to consider many different aspects of the situation—shape of weir, channel geometry, ratio of head to crest length—before classifying the weir as to the appropriate discharge equation and coefficient.

Because weir flow involves free curvilinear flow and the combined influence of several fluid properties, it is not subject to complete mathematical description. The most direct solution for the discharge function thus involves a combination of dimensional analysis and experiment. This approach was used to determine the equations for the different types of weirs.

The reliability of the computed discharge over a weir depends primarily upon selection of the proper discharge coefficient. For some dams this coefficient must be estimated by comparison with calibrated weirs of similar shape, for others it must be computed from curves of general relations based on laboratory studies. Information available at present provides coefficients for (1) sharp-crested weirs, either discharging freely or submerged, (2) broad-crested weirs not submerged, (3) ogee or design-head dams, submerged or not submerged, and (4) many irregular shapes described in Water-Supply Paper 200.

## Selection of Site

The two most important elements of a computation of flow over a weir, dam, or embankment are the head,  $h$ , and the discharge coefficient,  $C$ . However, if high velocity of approach makes the velocity head a substantial part of the total head, then the reliability of the entire measurement is subject to some question. This is true because at present there is no reliable method of determining  $\alpha$ , the energy head coefficient. Even if  $\alpha$  could be determined, by itself it may not be so important as the velocity distribution in the channel approaching the dam.

For some types of dams the discharge coefficient can be determined quite reliably. These dams are described in detail in other

parts of this manual. If the dam under study fits within the verified range of parameters, the discharge coefficient,  $C$ , can be selected with considerable confidence.

Thus, if the total head on the dam can be accurately determined and if the discharge coefficient is within the range for which adequate definition is available, then a reliable discharge can be computed for the dam.

## Field Procedures

Make a transit survey of floodmarks and accurate measurement of all pertinent features of the dam geometry as soon after the flood as possible. Do not use construction drawings without a thorough check of all important dimensions. Follow the methods of surveying described in chapter A1 by Benson and Dalrymple (1967). Record elevations of high-water marks, hubs, reference marks, and the crest of the weir to hundredths of a foot; record ground elevations to tenths of a foot.

## High-water profiles

Get high-water marks on both banks to define the headwater and the tail-water elevations. The head,  $h$ , on the dam is measured at a distance three to four times  $h$  upstream from the crest. Hence, it is most important that high-water marks be obtained in this area. Also, if the approach section is run at this same point, it is not necessary to correct for friction loss between the approach section and the dam.

Obtain elevation of the tail water for every computation of flow over a weir. It may be obvious in the field that the tail water was well below the crest of the dam, but always record the tail-water elevations in the field notes. If the tail-water is higher than the crest of the dam, determine tail-water profiles accurately because they may be used in the computation of discharge. Obtain tail-water elevations downstream from the energy recovery zone.

## Cross sections

Survey a cross section in the approach reach to define the approach velocity and the height of the weir or dam above the stream bed. Make the distance from the weir crest to the

cross section equivalent to three to four times the head,  $h$ . If preliminary observations indicate that the ratio of weir height to head is greater than 10, it is not necessary to define the cross section in detail.

If high-water marks cannot be found at the prescribed location, it may be necessary to obtain the cross section farther upstream. However, always obtain the section as close to the prescribed location as possible to avoid adjustments for friction loss.

## Dam details

### Crest shape

Define the shape of the dam crest in the direction of flow to hundredths of a foot and in considerable detail. The slope of the upstream face, the rounding of the upstream edge, the width of the crest, the roughness of the crest, and the slope of the downstream face are important in determining the discharge coefficient. The upstream edge of a dam as often found in the field has been battered by rocks and logs until it has a semirounded edge which is difficult to describe. The field engineer should realize that he may be the only man to see the dam and his description and measurements will form the basis for selecting a discharge coefficient. Hence, make every effort to describe completely the crest shape. If the shape of the crest varies along its length, detail every change in the shape. The shape of the dam normal to flow should, of course, be very accurately measured in both the horizontal and the vertical planes.

### Piers and abutments

Measure adequately the pier and abutment details and document them so that they can be exactly reproduced in plan view in the office. The radius of rounding of the upstream corner of the abutments is particularly important in the derivation of the discharge coefficient.

### Gates and flashboards

If the dam is equipped with control gates or flashboards, it is very important that the opening of the gates or the condition of the flashboards at the time of the peak discharge be ascertained. If the gates (tainter, drum, lift, roll, or other) in any way affected the flow,

obtain detail drawings of the gates. Get this information as soon as possible after the peak. If records of gate operation are not kept by the agency in charge of the dam, question the dam tender for this information. If a record of operations has been maintained, make a copy of this record, along with any pertinent comments, as part of the field notes.

### Debris

Debris lodging on piers, abutments, or the dam crest itself may have an appreciable effect on the head-discharge relationship. Here again, the quicker the dam can be inspected after the peak, the better. If at all possible, photograph conditions of the dam crest and piers as they were at the time of the peak. Lacking such a photograph, obtain a detailed description by the dam tender or maintenance crews.

### Water bypassing the crest of the dam

Determine the quantity of water bypassing the spillway crest at the time of the peak. This might be through a powerhouse, an irrigation canal, a public water supply, a fish ladder, or some other diversion. If the agency in charge of the dam has a record of its operations during the flood, obtain a copy of this record.

### Pictures

Pictures of the conditions of the dam crest or flow conditions at the time of the peak are very desirable. If the engineer himself cannot obtain these, he should question the dam tender or maintenance people to see whether they have taken any pictures at the time of the peak or shortly thereafter. At the time of the survey, obtain detailed stereophotographs of every pertinent feature of the dam. Closeup pictures of the dam crest (particularly the upstream edge), piers, abutments, gates, flashboards, and debris conditions are particularly important. Include a scale reference in each picture, such as a ruler or level rod.

### Office Procedures

Plot the plan, elevation, and cross-sectional views which show the geometry of the channel

and dam. Follow the computation procedures outlined in this report for the particular dam in question. Dams will be classified differently for different heads or shape parameters. For example, a broad-crested weir or a round-crested weir may act as a sharp-crested weir at certain heads. Some computations of flow over a dam are a cut-and-try process. That is, a discharge is assumed so that the velocity head in the approach section can be computed. Velocity head, plus the piezometric head, gives the total head used to compute the flow over the dam. This process is repeated until the assumed discharge checks the computed discharge within one percent. Other computations make use of the static head, with the effect of velocity of approach being taken into account in the discharge coefficient. Hence, the user of this report is cautioned to make certain which head, static ( $h$ ) or total ( $H$ ), is applicable for the dam being studied.

### Piers and abutments

Compute the effective length of the crest by deducting the width of piers from the total length. Neglect other minor effects of piers on the discharge coefficient in the computation of discharge. These minor effects, which include contraction of the flow and changes in pressure in the lower nappe, may slightly increase or decrease the discharge coefficient at a given head. However, data to define adequately the effect of piers on the discharge coefficient are not available.

Abutments which contract the flow section have an appreciable effect on the discharge coefficient. The adjustment factor used to account for this effect is a function of the degree of contraction and the radius of rounding of the upstream corner.

### Weirs not level

If the crest of the dam or embankment is not truly horizontal, but is a little inclined ( $h_a \approx 1.5h_b$ ), the discharge may be closely approximated by the use of the average crest elevation to obtain  $h$  in the ordinary weir formula. But more precisely, the correct discharge should be computed by the formula below. This formula is applicable to any dam

that has a uniformly sloping crest, if the value of  $C$  remains constant from one end of the dam to the other. The equation is given in terms of the static head,  $h$ . The total head,  $H$ , should be substituted for  $h$  if the discharge coefficient for a particular dam is referred to that head.

$$Q = \frac{2Cb}{5(h_b - h_a)} (h_b^{5/2} - h_a^{5/2}), \quad (1)$$

where

- $Q$  = discharge,
- $C$  = discharge coefficient,
- $b$  = length of dam normal to flow,
- $h_a$  = static head referred to high end of weir,  
and
- $h_b$  = static head referred to low end of weir.

### Weirs with curved or angled crest in plan

If the length of the crest greatly exceeds the width of the channel, the effective length of the crest is uncertain. For high dams with low-approach velocity the entire length should be considered effective. The correct procedure to use for low dams with high-approach velocity has not been defined.

### Determination of weir height

The height of the weir,  $P$ , is an essential element in the selection of the discharge coefficient. It is computed as the height of the weir above the average streambed elevation in a  $b$  width of the approach section taken 3-4  $h$  upstream from the weir. The  $b$  width represents a projection of the length of the spillway normal to the flow.

### Rectangular Sharp-Crested Weirs

Truly sharp-crested weirs are very seldom used in field installations on natural channels where floodflows occur. Yet many types of dam crests in the field are for the most part sharp crested or act as such.

Flashboards which form the effective crest may usually be treated as sharp-edged weirs if they are high enough or are placed so that the upstream edge of the dam does not materially influence the flow pattern. A dam with

a sharp upstream corner and level crest may act as a sharp-crested weir if the lower nappe springs clear of the downstream edge. With no appreciable velocity of approach and with complete aeration downstream, the lower nappe will clear if the head is greater than about 1.5 times the length of crest in direction of flow. Many round-crested dams are designed to fit the lower nappe profile of a sharp-crested weir at a given discharge, and the analysis of their discharge characteristics is based on data for sharp-crested weirs. For these reasons information on sharp-crested weirs is presented in detail in this report.

### Basic equation

Flow over a rectangular sharp-crested weir is illustrated on figure 1. The discharge equation for this type of weir as used in this report is:

$$Q = Cbh^{3/2}, \quad (2)$$

where

- $Q$  = discharge,
- $C$  = discharge coefficient,
- $b$  = width of weir crest normal to flow,  
excluding width of piers, and
- $h$  = static or piezometric head on a weir,  
referred to the weir crest.

### Discharge coefficients

Information on discharge coefficients for rectangular sharp-crested weirs is available from the investigations of the Bureau of Reclamation (1948), Kindsvater and Carter (1959), and many others. These investigations show that the coefficient for free discharge is a function of certain dimensionless ratios which describe the geometry of the channel and the weir,

$$C = f\left(\frac{h}{P}, \frac{b}{B}, E\right),$$

where  $E$  is the slope of the weir face; the other variables are depicted on figure 1.

The relation between  $C$ ,  $h/P$  and  $E$  for weirs with no side contraction ( $b/B=1.0$ ) is shown on figure 2. The coefficient is defined in the range of  $h/P$  from 0 to 5. The value of the coefficient becomes uncertain at higher values of  $h/P$ .

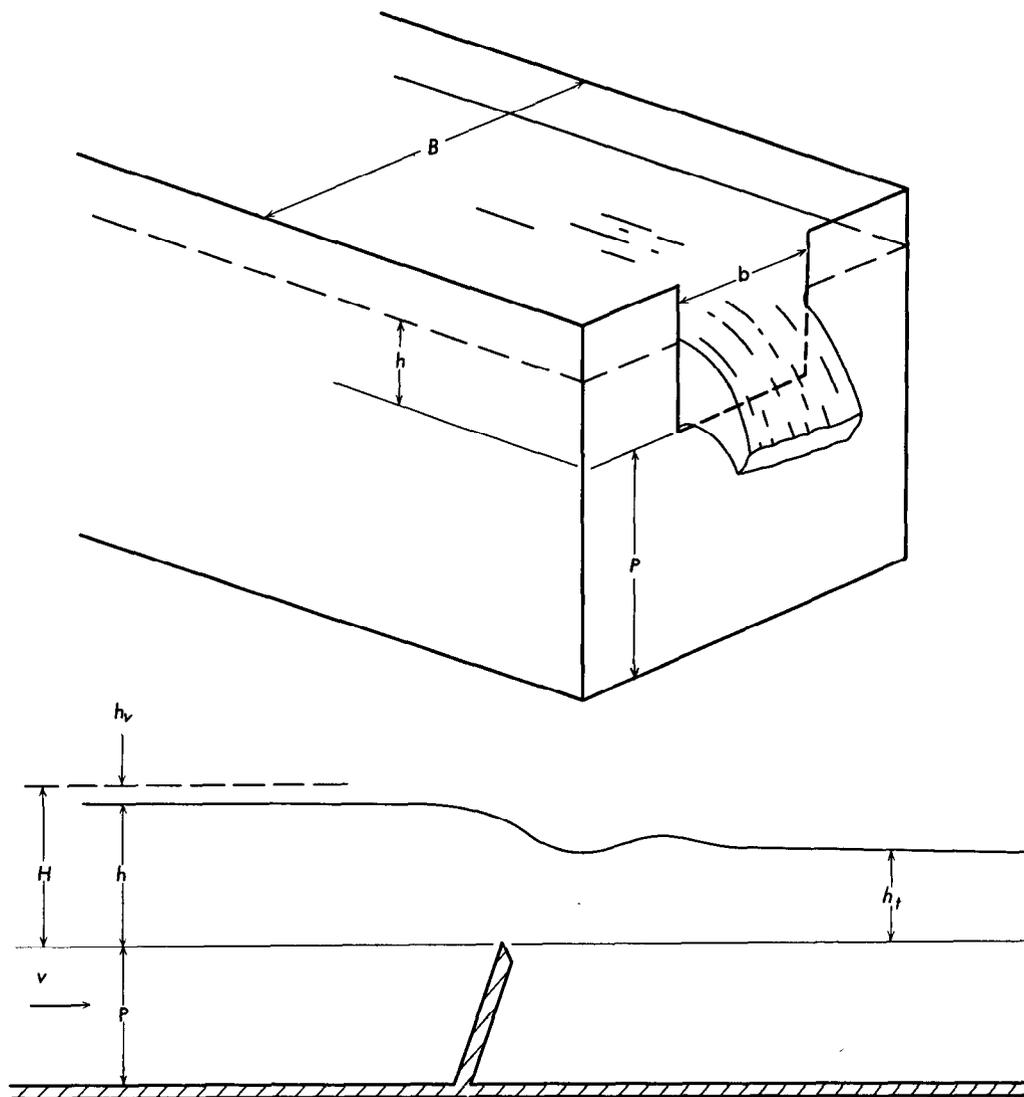


Figure 1.—Definition sketch of a sharp-crested weir.

Side contractions reduce the effective length of the weir crest. This effect is accounted for by multiplying the value of  $C$  from figure 2 by a correction factor  $k_c$ . This factor is a function of  $b/B$  and  $h/P$ , and the relationship for a square-cornered entrance is defined on figure 3. If the abutment corners are rounded with radius  $r$  and  $r/b > 0.12$ , assume that  $k_c = 1.00$ . For lesser values of  $r/b$ , interpolate between these two values.

### Submerged sharp-crested weirs

If the elevation of the tail water is above the elevation of the crest of the weir, the effect of

submergence on the head-discharge relation for free flow must be considered. Many investigations of flow over submerged weirs have shown that the ratio,  $k_t$ , of discharge under submerged conditions to that for free flow can be expressed as follows:

$$k_t = f\left(\frac{h_t}{H}, \frac{H}{P}\right),$$

where

$h_t$  = height of tail water above weir crest,  
 $H$  = height of headwater total energy level above weir crest, and  
 $P$  = height of weir.

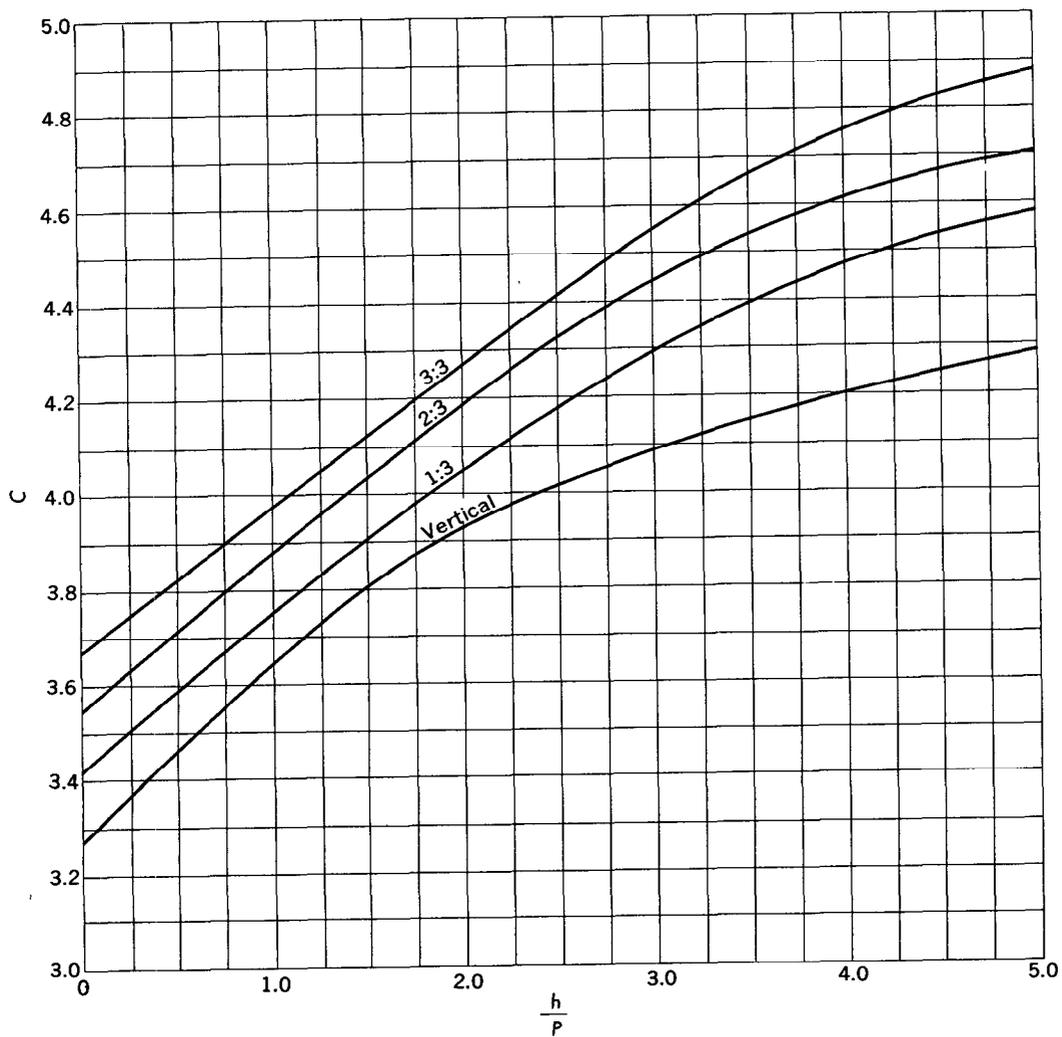


Figure 2.—Discharge coefficients for full width, vertical, and inclined sharp-crested rectangular weirs.

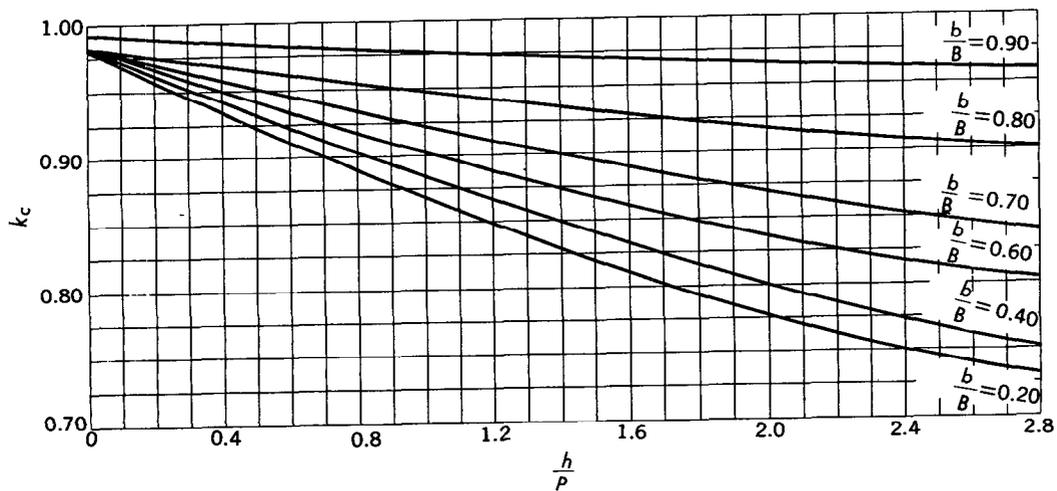


Figure 3.—Definition of adjustment factor,  $k_c$ , for contracted sharp-crested weirs.

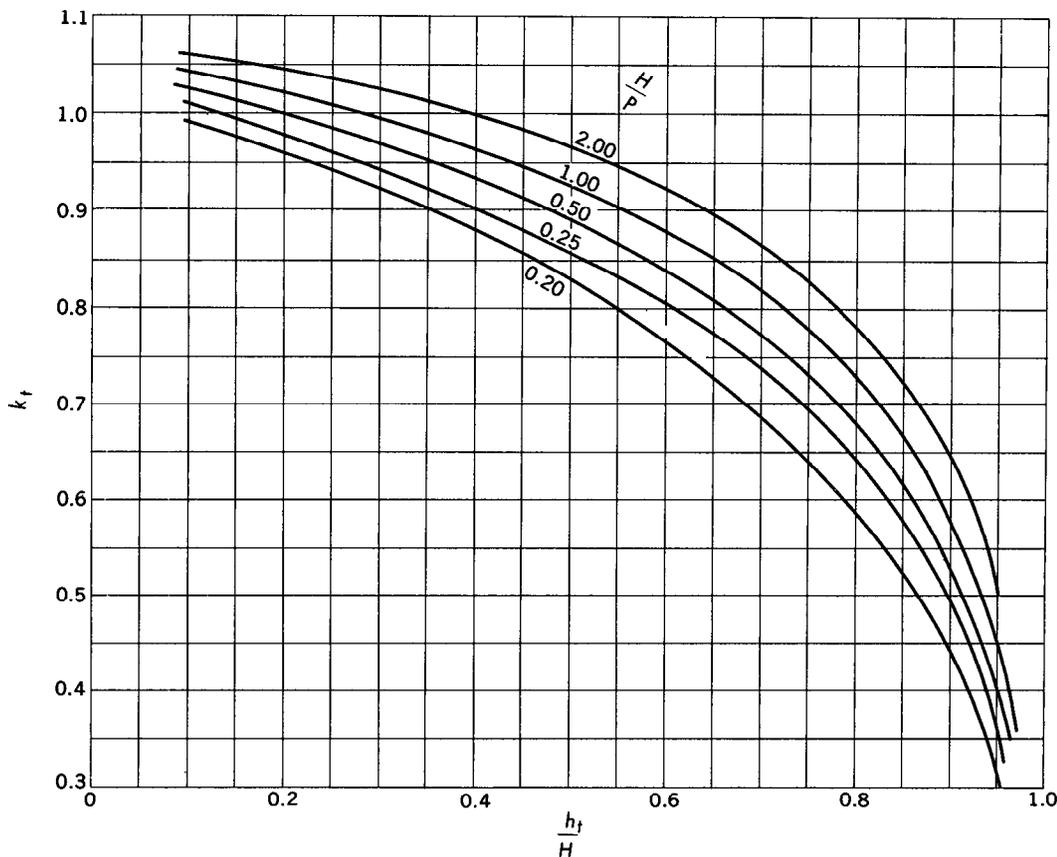


Figure 4.—Definition of adjustment factor,  $k_t$ , for submerged sharp-crested rectangular weirs.

Vennard and Weston (1943) used the work of several investigators to define the relationship shown in figure 4. This relationship may be used to compute the discharge over submerged weirs after the discharge for free flow conditions is determined by using the equation and coefficients given in the previous sections.

## Broad-Crested Weirs

### Definitions

Pertinent features of the geometry and flow pattern for a broad-crested weir are illustrated on figure 5. The weir has a nearly horizontal crest of finite length,  $L$ , in the direction of flow. A zone of curvilinear flow occurs near the upstream end of the weir, and separation occurs unless the upstream corner is rounded. If the weir is sufficiently long, the flow may become parallel to the crest.

The term "broad-crested weir" is generally poorly defined; usually, this weir has been classified only with respect to the geometry of the structure itself. Almost universally, a weir is called broad-crested if it has a nearly horizontal crest of finite length in the direction of flow. This definition is not entirely adequate, because at sufficiently high head-to-length ratios the nappe tends to spring clear of the weir crest, and the structure no longer performs as a broad-crested weir. At the opposite extreme, for a very small head-to-length ratio, the weir crest becomes a reach of open channel in which frictional resistance predominates, and for which the discharge is more properly evaluated by one of the open-channel flow formulas than by a weir formula. It is thus clear that any definition of a broad-crested weir must include the head acting on the weir.

The point at which the nappe becomes detached at the upstream crest entrance and

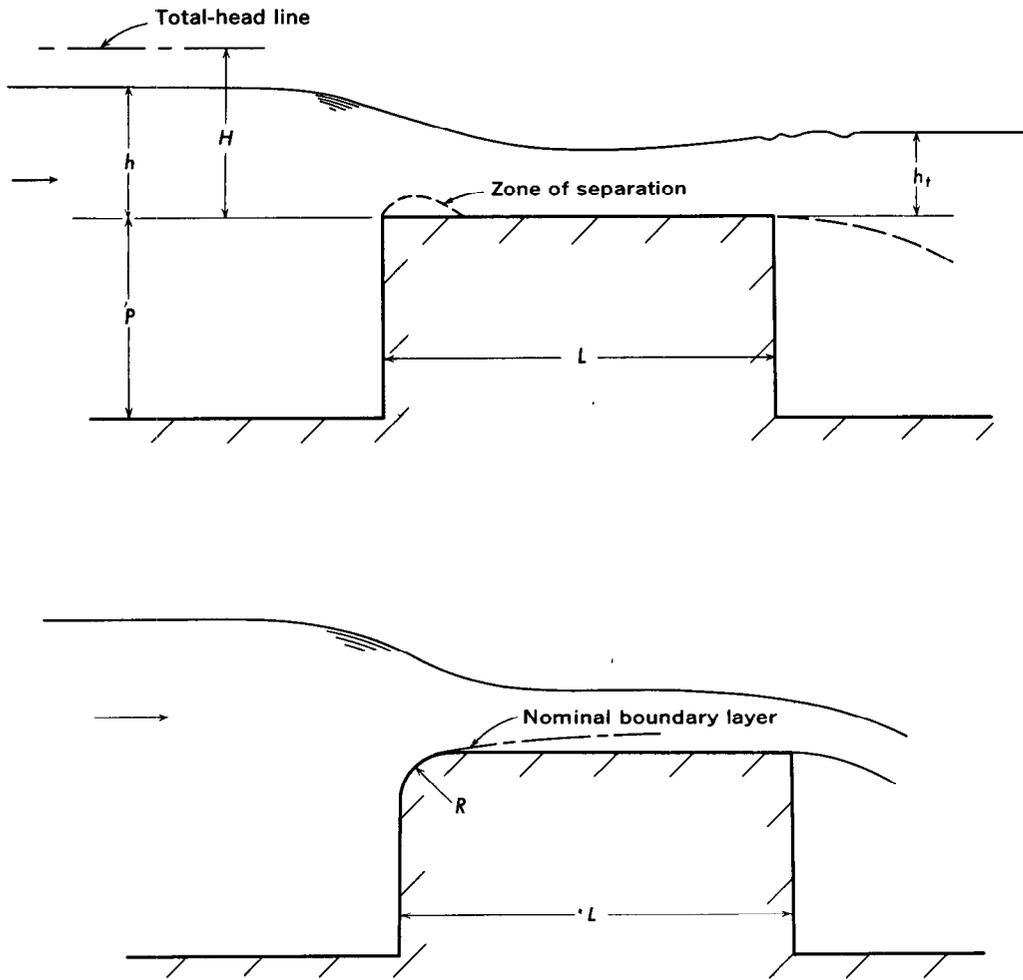


Figure 5.—Definition sketches of broad-crested weirs with vertical faces and horizontal crest.

springs clear of the weir crest varies somewhat, depending upon the completeness of aeration of the space beneath the downstream nappe, the  $h/P$  ratio, and the sharpness of the entrance. The tests made by Bazin indicate that it may occur at an  $h/L$  as low as 1.5, or as high as 2.3, for aerated weirs.

The minimum  $h/L$  ratio for which the lower nappe will clear the downstream corner of a broad-crested weir is shown on figure 6. If the nappe clears the downstream corner, the weir should be treated as a sharp-crested weir; otherwise it is classified as a broad-crested weir, provided the ratio  $h/L$  is greater than 0.10. If  $h/L$  is less than 0.10, use figure 23.

### Basic equation

The discharge equation for broad-crested weirs is more conveniently expressed in terms of the total energy head,  $H$ . The discharge equation is:

$$Q = CbH^{3/2}, \quad (3)$$

where

$Q$  = discharge,

$C$  = a coefficient of discharge,

$b$  = width of the weir normal to the flow, excluding width of piers, and

$H$  = total energy head ( $h + V_1^2/2g$ ) referred to the crest of the weir, and  $V_1$  is the

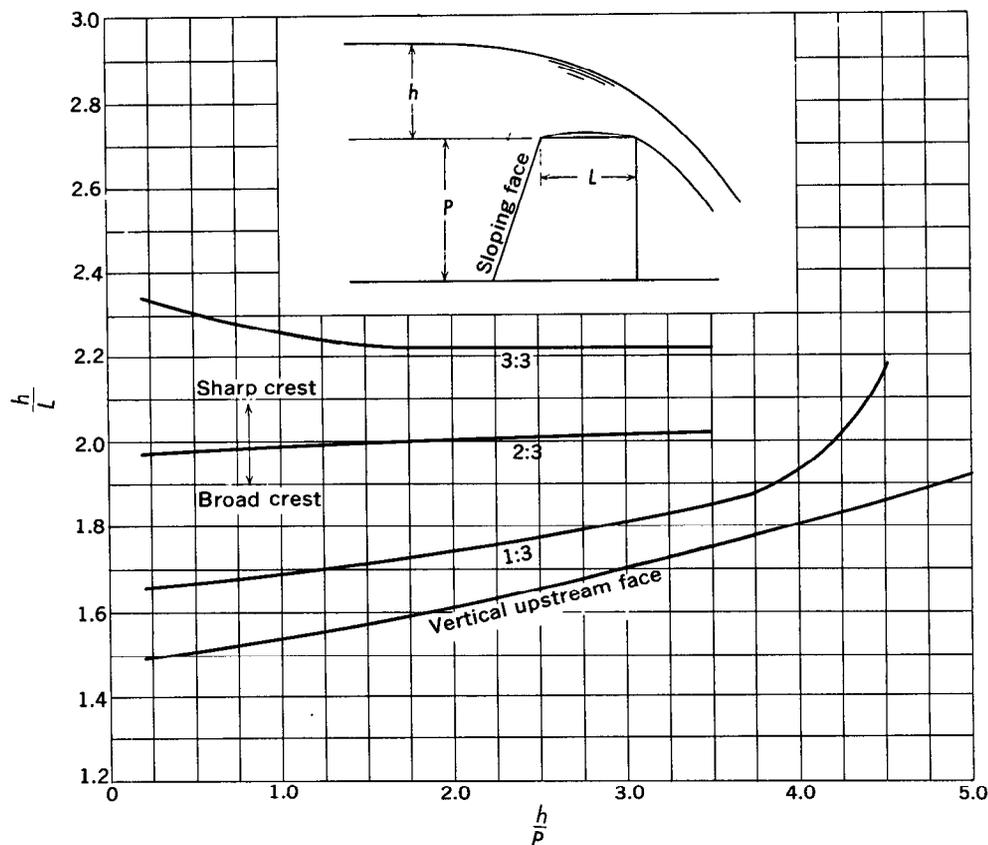


Figure 6.—Minimum  $h/L$  ratio for which the lower nappe will clear the downstream corner of a broad-crested weir.

mean velocity at the approach section to the weir.

### Discharge coefficients

The discharge coefficient in equation 2 for free discharge over the weir is a function of the following dimensionless ratios,

$$C = f\left(\frac{h}{L}, \frac{R}{h}, E_1, E_2, \frac{b}{B}, \frac{r}{b}\right)$$

where  $R$  is the radius of rounding of the upstream vertical face at the entrance,  $r$  is the radius of rounding of the side abutment, and  $E_1$  and  $E_2$  are the slope of the upstream and downstream face of the weir. The slope is defined as the ratio of the horizontal to vertical distance.

The relation between  $C$ ,  $h/L$  and  $E_1$  for the condition  $E_2$  equal to or less than 1:1 and  $b/B$  equal to 1 is shown on figure 7.

If the slope of the downstream face of the weir is flatter than 1:1, the values of  $C$  from figure 7 must be multiplied by a factor,  $k_s$ , from the following table:

$h/L$	Downstream slope			
	2:1	3:1	4:1	5:1
0.1	1.00	1.00	1.00	1.00
.4	1.00	1.00	1.00	1.00
1.0	.98	.96	.95	.94
2.0	.98	.94	.91	.90

If the upstream weir face is vertical and the entrance corner is rounded, the value of  $C$  from figure 7 must be multiplied by a factor,  $k_R$  from the following table:

$\frac{R}{h}$	$k_R$	$\frac{R}{h}$	$k_R$
0	1.00	0.08	1.05
.02	1.01	.10	1.06
.04	1.03	.12	1.08
.06	1.04	.14	1.09

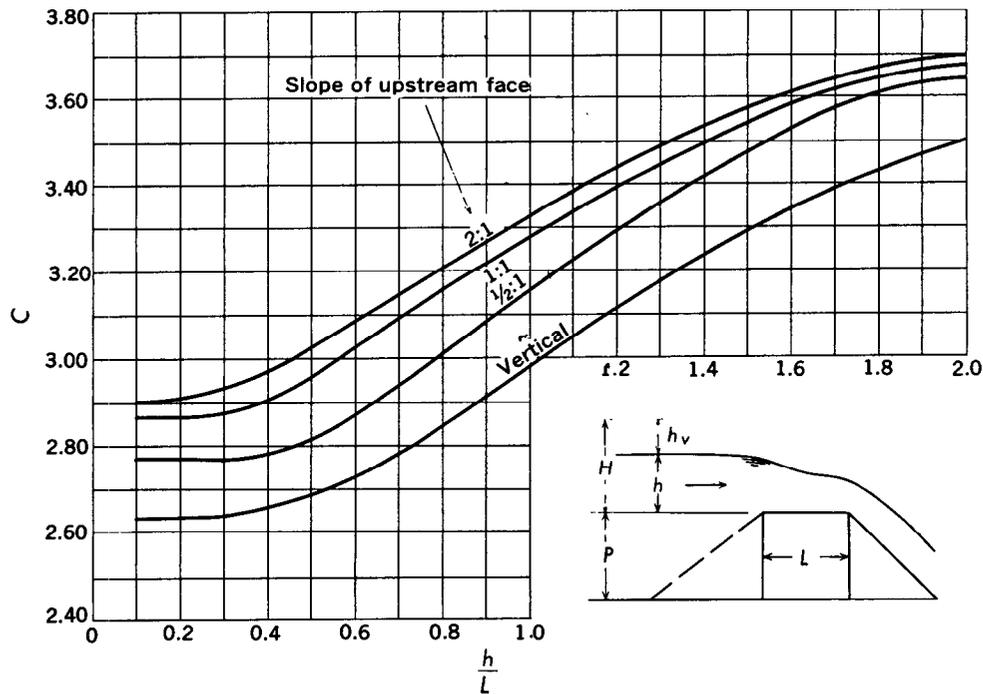


Figure 7.—Coefficients of discharge for full width, broad-crested weirs with downstream slope  $\cong 1:1$  and various upstream slopes.

Side contractions (abutments) reduce the effective length of the weir crest. This effect is accounted for by multiplying the value of  $C$  determined by the procedures given above by a correction factor,  $k_c$ . If the abutment corners are square, use the factor obtained from figure 3. If the abutment corners are round and  $r/b > 0.12$ , assume that  $k=1.00$ . For lesser values of  $r/b$ , interpolate between these two values.

### Submerged broad-crested weirs

The degree of submergence of a broad-crested weir is defined by the ratio of the downstream static head  $h_1$  to the upstream static head,  $h$ . Submergence has no effect on the discharge coefficient if the submergence ratio  $h_1/h$  is less than 0.85. At larger ratios submergence has an appreciable effect, but test data to quantitatively define the effect are not available.

### Round-Crested Weirs

Round-crested weirs are defined as weirs with crests that are smooth single-curved (cylindrical) surfaces. The longitudinal pro-

file of the crest generally is not a circular arc, as the name may imply; often it is a complex curve which cannot be described in simple geometric terms. The crests of overfall spillways of most modern high dams, which have the form of the lower surface of the nappe from a full-width thin-plate weir, are special examples of round-crested weirs. These are commonly called ogee or design-head dams.

The profile geometry of round-crested weirs shows that they are in a class intermediate between thin-plate weirs and broad-crested weirs. The crest is long enough (in the direction of flow) to provide support for the nappe through the control section, but it is short enough that the flow over the crest is markedly curvilinear. The fixed lower boundary of the nappe is therefore subject to much variation in pressure; for any given head the pressure may vary considerably from point to point, and at any given point the pressure varies appreciably with the head.

The geometry and flow pattern for a simple round-crested weir are illustrated on figure 8. The intersection of the curvilinear crest and the vertical (or inclined) face of the weir is

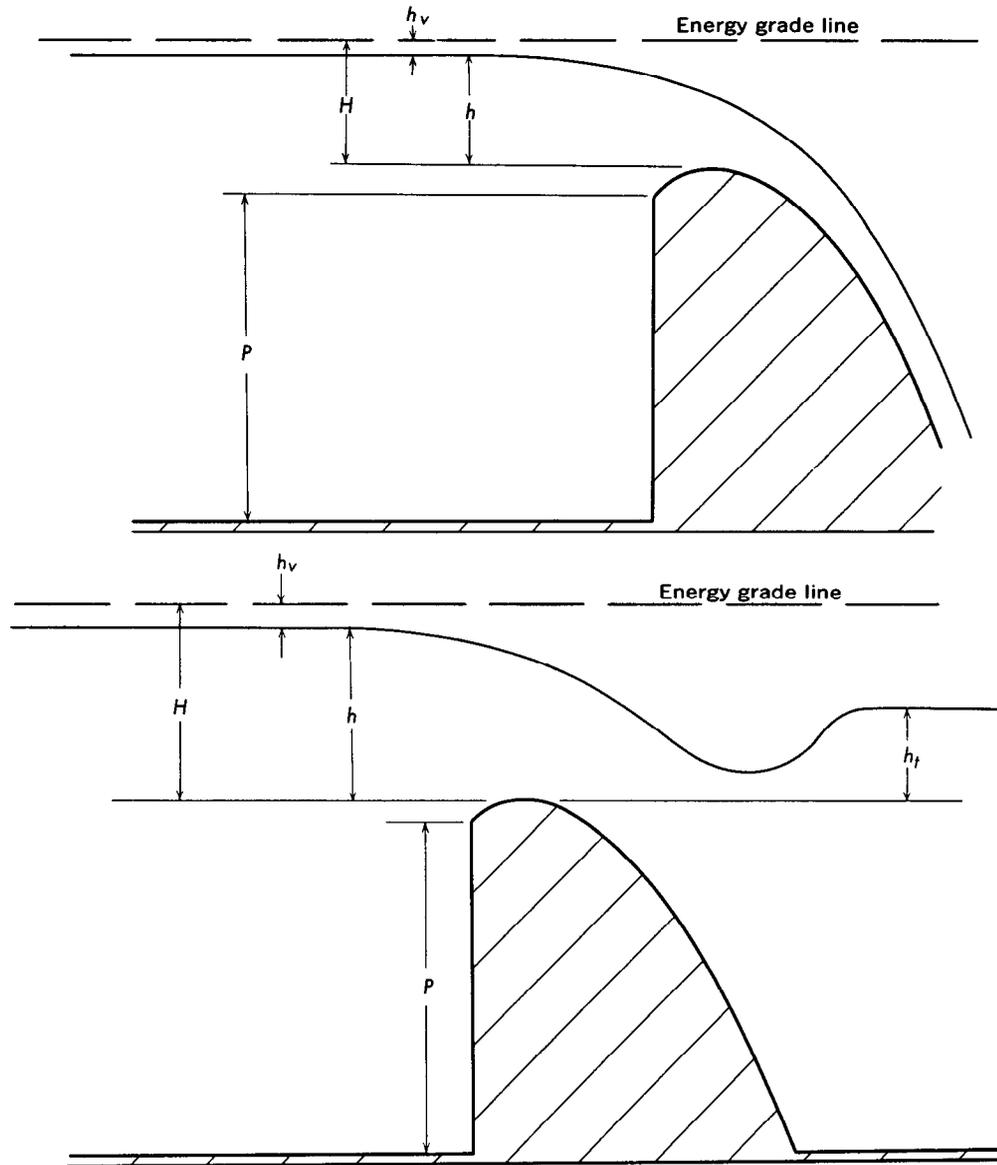


Figure 8.—Definition sketches of a nappe-fitting design-head dam with free flow and with submerged flow.

termed the "spring point" and the height,  $P$ , of the weir is measured to this point. The head on the weir is referenced to the highest point on the crest.

The height,  $P$ , of the weir above the floor of the approach channel is an important property of the profile because, together with the head, it is a measure of the degree of contraction in a vertical plane imposed on the flow by the boundary. This, in turn, affects the pressure distribution along the crest and

in the control section, for the following reason: The height of the weir relative to the head governs the trajectory that the particles in the lowermost filaments of the nappe would take if they were "free" (as in flow over a ventilated thin-plate weir). That is, the "rise" of the free nappe is controlled by the degree of vertical contraction of the boundaries, just as the contraction of the jet from an orifice is controlled by the relative magnitudes of pipe and orifice diameters. Within limits, the

higher the weir is for a given head, the greater the rise. Thus, if a round-crested weir has a crest which fits exactly a free nappe for a certain head, the pressure along the crest is substantially zero (atmospheric). If, with no change in head or in crest conformation, the height of the weir were to be made smaller, the solid crest would protrude above the profile of the nappe from a thin-plate weir of comparable height. The pressure along the solid crest would no longer be zero. Positive pressure would exist because the crest would deflect the stream slightly upward from its "free" position. The increase of pressure in the control section must result in a decrease in discharge coefficient.

The inclination of the upstream face of the weir likewise influences the discharge through its influence on the contraction of the nappe. As a general rule, the flatter the slope of the upstream face the lower the free-nappe profile will be.

Round-crested weirs comprise an unlimited variety of specific crest profiles, even within the limitation that no profile may include local curvature so great that the nappe separates from the crest. Because viscous effects are considered to be negligible, the boundary geometry uniquely determines the flow pattern over the crest. Thus, for each specific boundary profile there is a unique flow pattern for any given head, with unique velocity and pressure distributions in the nappe which determines the discharge coefficient.

### Basic equation

The discharge equation for round-crested weirs as used in this manual is referenced to the piezometric head  $h$ . The equation is

$$Q = Cbh^{3/2}, \quad (2)$$

where

$Q$  = discharge,

$C$  = coefficient of discharge,

$b$  = length of the crest normal to the flow excluding piers, and

$h$  = static head referred to the crest of the dam.

### Discharge coefficients

The discharge coefficient for round-crested weirs is a function of both weir geometry and head. Because of the endless variety of weir shapes, however, no general solution for the discharge coefficient is available. One of the following methods is generally used to establish discharge coefficients for a particular weir.

1. Results of model tests for the particular dam used in the measurement.
2. Comparison with other crest for which the discharge coefficient is known.
3. Nappe-fitting method.
4. Index-measurement method.

Model tests are seldom available except for very large dams. However, this possibility should be investigated. The owners of the dam will usually provide this information if it is available.

Perhaps the oldest most widely used procedure for estimating a discharge coefficient is to compare the profile of the weir in question with the profiles of others whose calibrations are known and to use the head-discharge coefficient relationship of the one which bears the closest resemblance. For similitude of the two flows, it is necessary that reasonably good correspondence of the crest profiles be obtained, especially from about the "spring" point to the crown; that a proportional, not equal, value of the head be used; that the relative heights of the weirs be not greatly different, especially if one weir is low; and that the width-contraction aspects of the calibrated weir be reasonably similar to those of the weir in question. Often, some important aspects of the geometry are not known, either for one weir or the other, and are necessarily ignored. An extensive catalog of weir calibrations which included all important data, systematically arranged, would improve the status of this method, but no such catalog is available.

The nappe-fitting method gives good results if the crest of the weir follows the form of the lower nappe from a full-width thin-plate weir. This method is explained in detail in the following section.

The index-measurement method depends on the availability of at least one discharge

measurement to define the coefficient at one value of head. It is most useful for crests which have an odd shape. The method is discussed in detail on page 23.

All four of the methods provide discharge coefficients for free-flow over a full-width weir. The effect of side contractions and submergence are considered separately.

#### Nappe-fitting method

Nappe fitting is the comparison of the profile of the crest with the profile of the lower nappe of sharp-crested weirs. The head on the sharp-crested weir which produces a lower nappe that coincides with the crest profile is called the reference head,  $h_s$ . This reference head and the corresponding coefficient for sharp-crested weirs may be used to determine the corresponding discharge coefficient for the round-crested weir at this particular head. Use can then be made of defined relationships to synthesize the entire head-coefficient curve of the round-crested weirs.

The nappe-fitting technique for determining the discharge coefficient for round-crested weirs was first presented by Borland (1938). Additional contributions were made by Bradley (1952) and Kirkpatrick (1957). The relationships presented in this report are based on a comprehensive study of existing information by Jones.<sup>1</sup>

Much of the basic information used in these studies was obtained by the Bureau of Reclamation.

#### Definitions for reference weir

The reference weir and corresponding symbols are illustrated on figure 9. The reference head,  $h_s$ , produces a lower nappe profile which rises a distance,  $E$ , from the spring point to the apex of the lower nappe. The design heads,  $h_0$  and  $H_0$ , are measured from the apex which coincides with the maximum point on the crest of a round-crested nappe-fitting weir. The design head,  $h_0$ , is equal to  $h_s - E$ . The height of the weir is measured to the top of the sharp-crested weir or to the spring point of the round-crested weir. The coordinates of the lower nappe profile are given in tables 1-4 in dimensionless

form as  $x/h_s$  and  $y/h_s$  as a function of  $h_s/P$ . The origin of the coordinates is at the spring point. The reference weir is used to determine the design head on a particular dam which leads to definition of the head-discharge coefficient relationship.

#### Procedure for determining design head

The following procedure may be used to determine the design head on a round-crested weir:

1. Measure the height,  $P$ , of the actual weir.
2. Assume a value of  $h_s$  and compute the ratio  $h_s/P$ . A good first approximation of  $h_s$  can be obtained by plotting the profile of the dam crest on an overlay, to the same scale as that of figures 10-13, and then using the overlay in conjunction with these figures to estimate  $h_s$ .
3. Plot the coordinates of the lower nappe corresponding to the assumed  $h_s$  by converting the appropriate data from table 1 to dimensional values of  $x$  and  $y$  for the assumed  $h_s$ .
4. Plot the profile of the actual weir on an overlay sheet to the same scale as in step 3 and compare the two profiles. To make this comparison, match the apex of the actual dam with the apex of the computed profile. Care should be taken that the best possible match is obtained from the spring point to a point somewhat beyond the crown and that the curvature of the profiles near the crown is the same. The origins (spring point) of the profiles need not coincide, but the coordinate system should remain parallel. Some examples of how crest profiles may be matched are shown on figure 14.

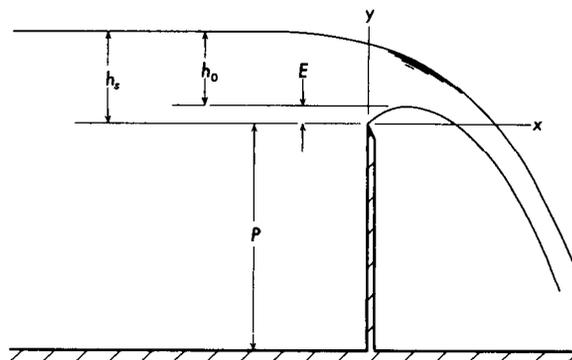


Figure 9.—Definition sketch of reference weir used in the nappe-fitting method.

<sup>1</sup> D. B. Jones, 1964, Discharge characteristics of round-crested weirs: written communication.

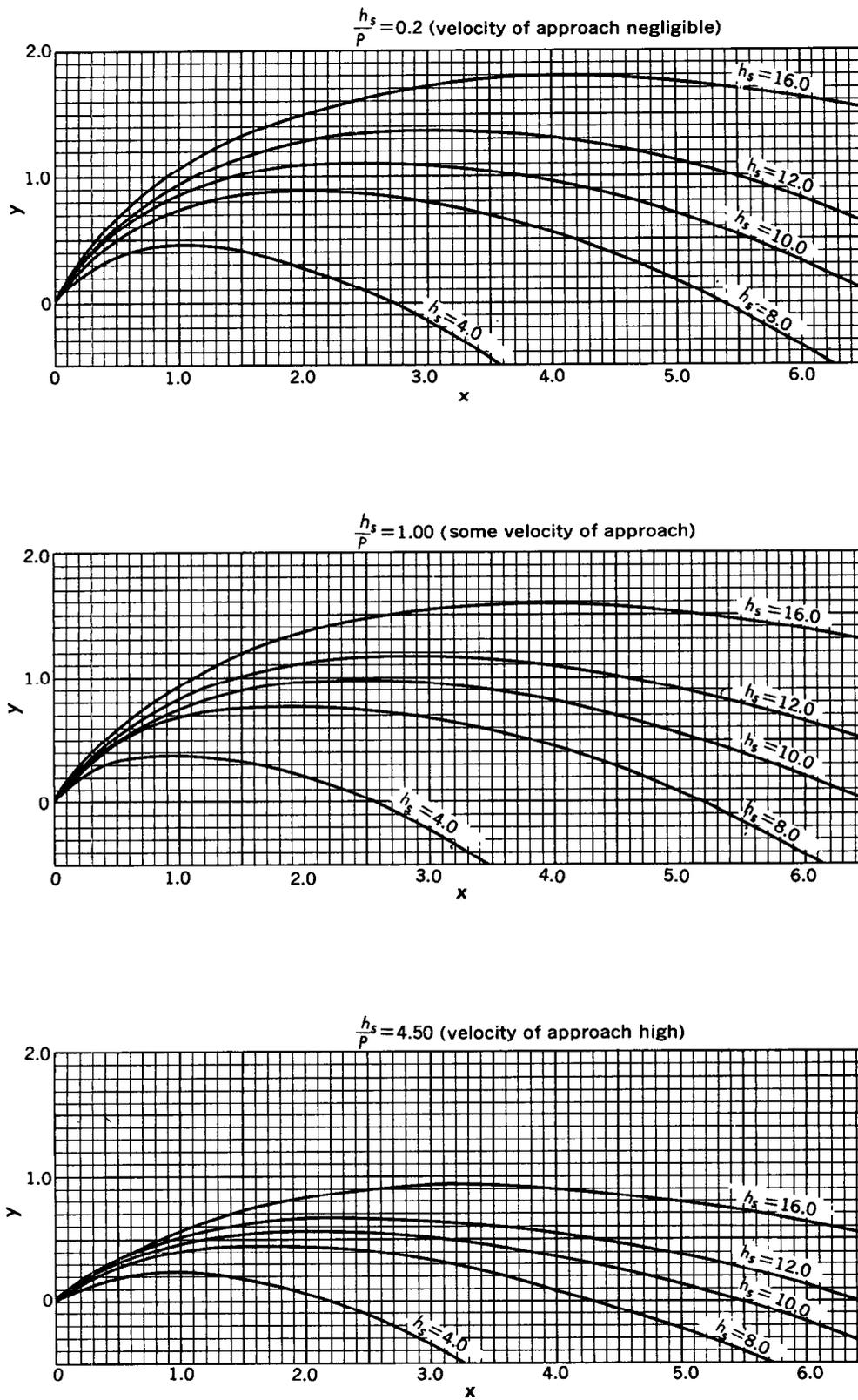


Figure 10.—Lower nappe profiles, sharp-crested weir, with upstream face vertical, from table 1.

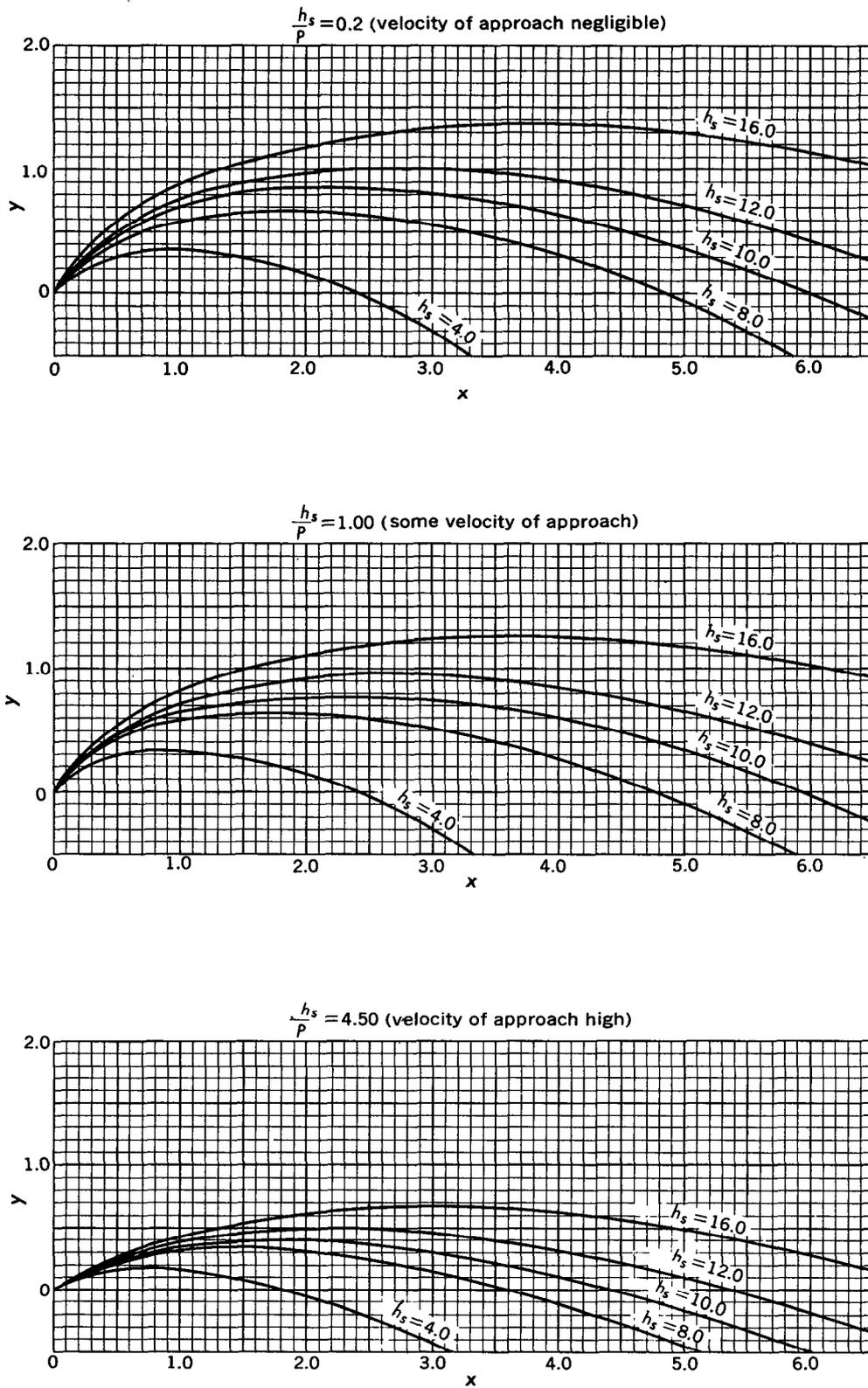


Figure 11.—Lower nappe profiles, sharp-crested weir, with upstream face sloped 1:3, from table 1.

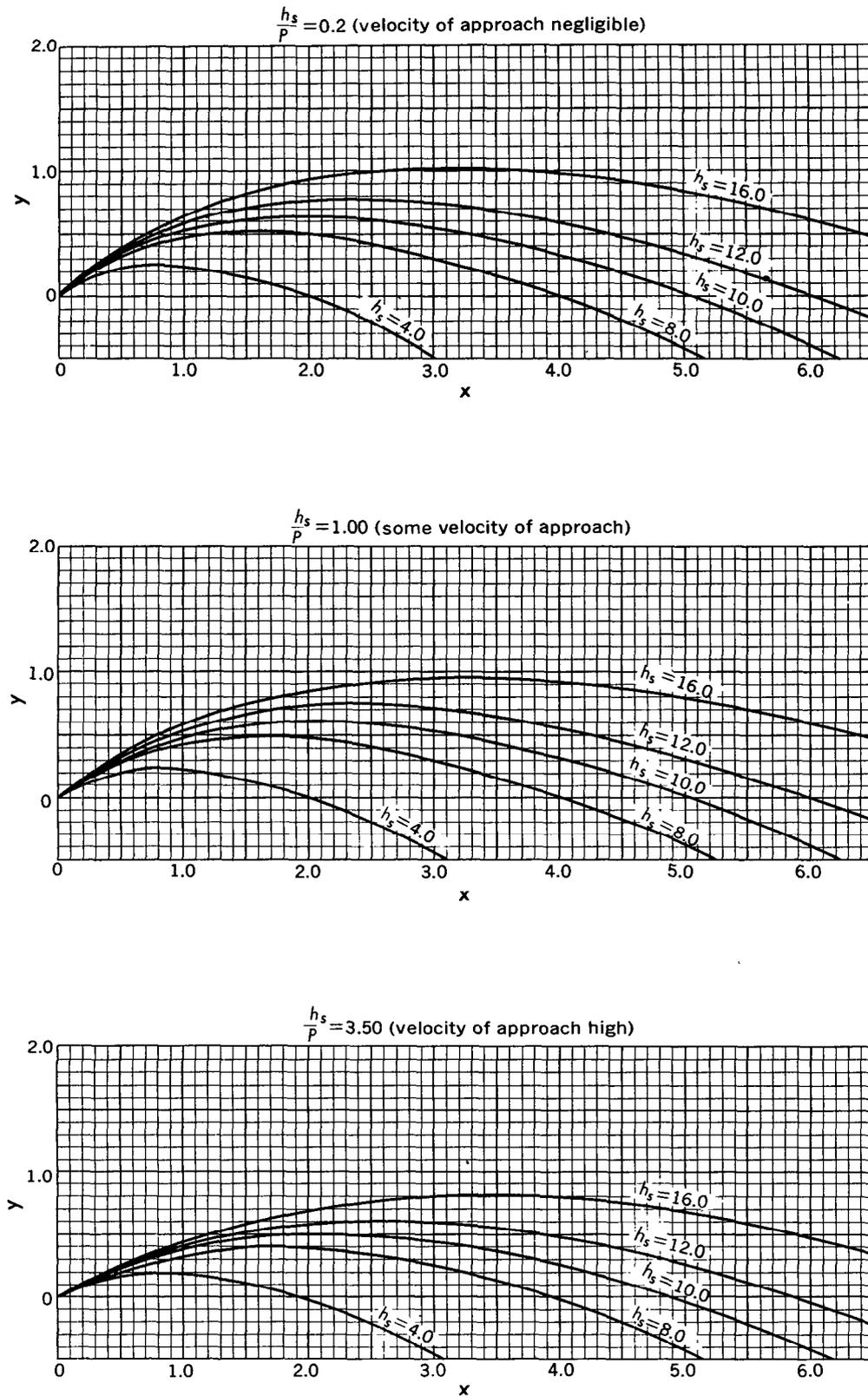


Figure 12.—Lower nappe profiles, sharp-crested weir, with upstream face sloped 2:3, from table 1.

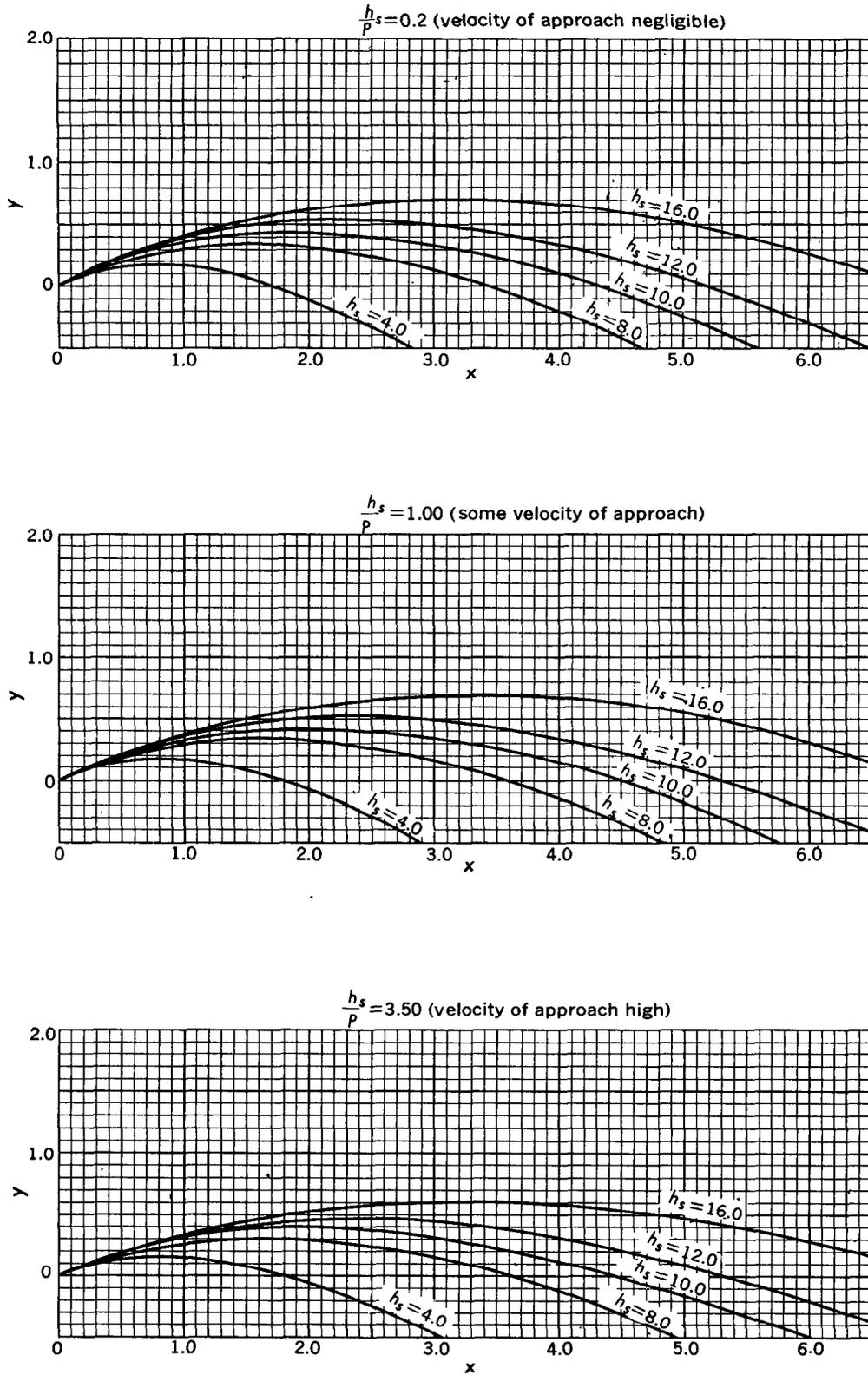


Figure 13.—Lower nappe profile, sharp-crested weir, with upstream face sloped 3 : 3, from table 1.

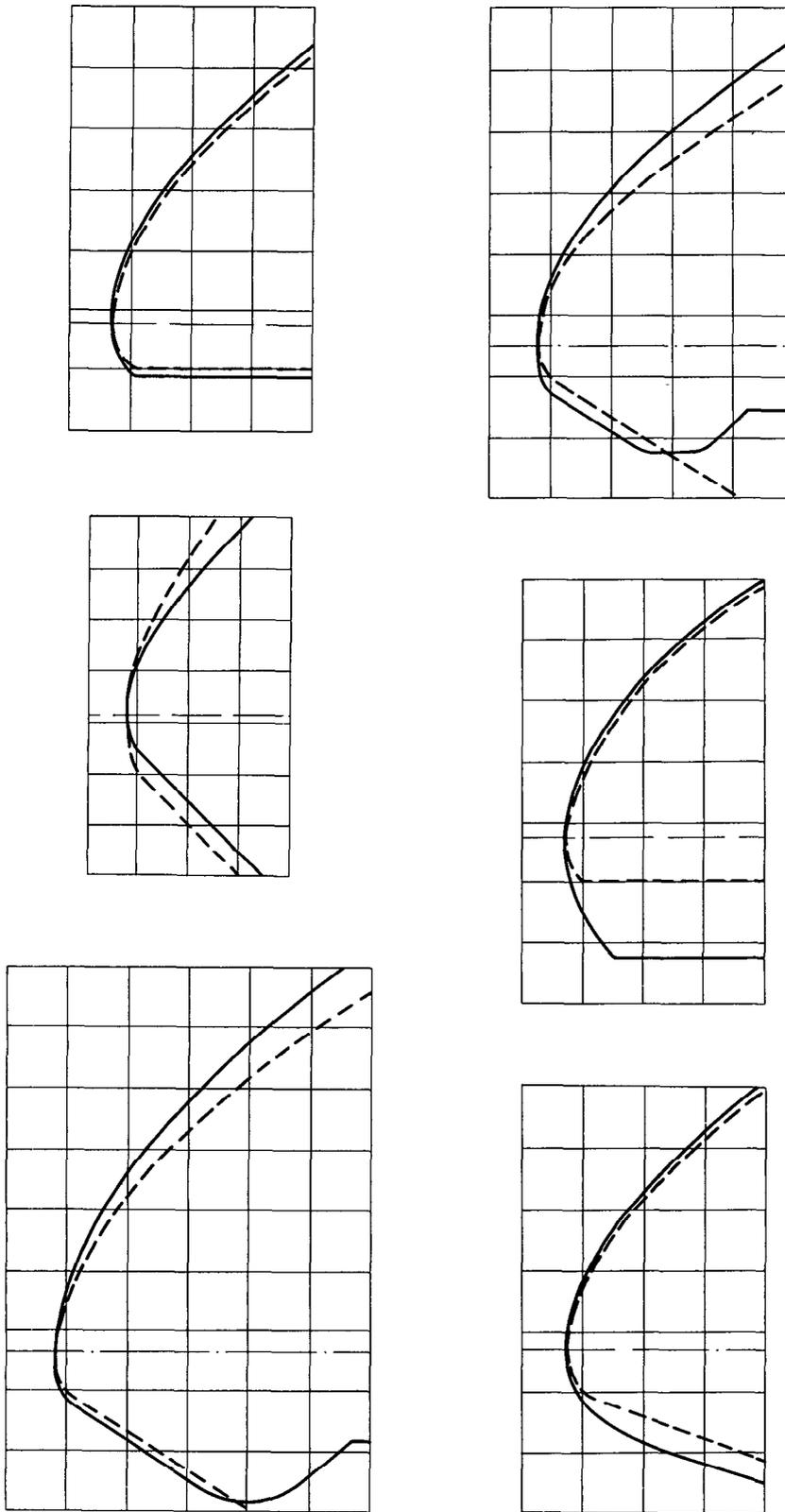


Figure 14.—Illustrations of how nappes-fitting profiles are fitted to actual dam-crest profiles.

If the profiles do not match, assume a new value of  $h_s$  and repeat the procedure, until the best possible fit is obtained. The value of  $h_s$  is not critical in determining the value of  $C$ ; thus values of  $h_s$  to the nearest foot are adequate.

5. After the value of  $h_s$  has been established by the above procedure, compute the value of

$E$  from the table used to define the coordinates of the lower nappe.  $E$  is the maximum value of  $y/h_s$  corresponding to the computed value of  $h_s/P$ . The design head,  $h_0$ , is equal to  $h_s - E$ . A slight discrepancy in the origin point for the two profiles generally exists after obtaining best agreement; this is neglected in computing  $h_0$  by the above procedure.

Table 1.—Coordinates of lower nappe for selected values of  $h_s/P$  for sharp-crested weir

$x/h_s$	$y/h_s$														
	0.01	0.10	0.20	0.40	0.60	0.80	1.00	1.20	1.50	2.00	2.50	3.00	3.50	4.00	4.50
Upstream face vertical															
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.01	.015	.015	.015	.014	.014	.014	.013	.013	.012	.011	.011	.010	.009	.008	.007
.02	.029	.028	.028	.027	.026	.025	.025	.024	.023	.021	.019	.019	.017	.016	.014
.03	.040	.039	.039	.038	.037	.036	.034	.033	.031	.028	.026	.025	.023	.022	.020
.04	.051	.050	.049	.047	.046	.045	.043	.041	.039	.035	.033	.031	.029	.027	.025
.05	.059	.058	.057	.055	.054	.052	.050	.048	.046	.042	.039	.037	.035	.032	.030
.06	.066	.065	.064	.062	.060	.058	.056	.054	.052	.047	.044	.042	.039	.037	.035
.07	.072	.071	.070	.067	.066	.064	.062	.060	.057	.052	.049	.047	.043	.041	.038
.08	.078	.077	.076	.074	.071	.069	.067	.065	.062	.057	.053	.050	.047	.044	.042
.09	.084	.082	.081	.079	.076	.074	.071	.069	.066	.061	.057	.054	.050	.047	.044
.10	.088	.086	.085	.083	.081	.078	.076	.073	.070	.064	.061	.057	.053	.050	.047
.11	.092	.090	.089	.087	.085	.082	.079	.077	.073	.067	.063	.059	.056	.052	.049
.12	.096	.094	.093	.091	.088	.085	.082	.080	.076	.070	.065	.061	.058	.054	.051
.13	.100	.098	.097	.094	.091	.088	.085	.083	.079	.073	.067	.064	.060	.056	.053
.14	.102	.100	.099	.097	.094	.091	.088	.085	.081	.075	.069	.065	.061	.057	.054
.15	.105	.104	.102	.099	.096	.093	.091	.088	.083	.076	.071	.066	.062	.058	.055
.16	.107	.106	.104	.101	.098	.095	.092	.089	.085	.078	.073	.067	.063	.059	.056
.17	.109	.108	.106	.103	.100	.097	.094	.090	.086	.079	.073	.068	.064	.059	.056
.18	.110	.109	.107	.104	.101	.099	.095	.092	.088	.080	.075	.069	.065	.060	.056
.19	.112	.110	.109	.106	.102	.100	.096	.093	.089	.081	.075	.070	.065	.060	.057
.20	.113	.112	.110	.107	.103	.101	.097	.093	.089	.082	.076	.070	.065	.060	.057
.21	.114	.112	.111	.108	.104	.101	.098	.094	.090	.083	.076	.070	.065	.060	.057
.22	.114	.113	.111	.108	.105	.102	.098	.094	.090	.083	.077	.070	.065	.060	.056
.23	.114	.113	.111	.108	.105	.102	.099	.094	.091	.083	.076	.070	.065	.060	.056
.24	.114	.113	.111	.109	.106	.103	.099	.095	.091	.083	.076	.070	.065	.059	.056
.25	.114	.113	.111	.109	.106	.103	.099	.094	.091	.083	.075	.069	.064	.059	.055
.26	.114	.113	.111	.109	.105	.102	.099	.094	.090	.083	.075	.068	.064	.059	.054
.27	.114	.113	.111	.108	.105	.102	.098	.094	.090	.083	.075	.068	.063	.058	.053
.28	.114	.113	.111	.108	.105	.102	.098	.093	.089	.081	.073	.067	.062	.057	.052
.29	.113	.112	.110	.108	.104	.101	.097	.093	.088	.081	.073	.066	.061	.056	.051
.30	.113	.112	.110	.107	.104	.101	.096	.092	.087	.080	.072	.066	.060	.055	.050
.31	.112	.110	.109	.106	.102	.099	.095	.091	.087	.079	.071	.064	.058	.054	.049
.32	.111	.109	.108	.105	.102	.098	.094	.090	.085	.077	.070	.063	.057	.052	.048
.33	.110	.108	.107	.104	.100	.097	.093	.089	.084	.076	.068	.062	.056	.051	.046
.34	.109	.107	.106	.103	.099	.096	.092	.088	.083	.075	.066	.060	.054	.050	.045
.36	.107	.105	.103	.100	.096	.092	.088	.084	.080	.071	.064	.057	.052	.046	.042
.38	.104	.102	.100	.096	.093	.089	.085	.081	.077	.068	.061	.054	.048	.043	.038
.40	.100	.098	.096	.093	.089	.085	.081	.077	.073	.064	.057	.051	.044	.039	.035
.42	.095	.093	.091	.088	.084	.081	.077	.073	.068	.060	.052	.046	.040	.035	.031
.44	.091	.089	.087	.083	.079	.076	.072	.068	.064	.055	.048	.042	.036	.030	.026
.46	.085	.083	.081	.078	.074	.071	.067	.063	.059	.051	.043	.037	.031	.026	.021
.48	.079	.078	.076	.072	.069	.065	.062	.058	.053	.045	.038	.032	.026	.021	.017
.50	.074	.072	.070	.067	.063	.059	.056	.052	.048	.039	.032	.026	.020	.015	.011
.52	.068	.066	.064	.061	.056	.053	.049	.046	.041	.033	.026	.020	.015	.009	.006
.54	.059	.058	.056	.053	.050	.046	.043	.039	.035	.027	.019	.014	.008	.003	.000
.56	.052	.050	.048	.046	.042	.039	.036	.032	.028	.020	.013	.006	.002	-.003	-.005
.58	.043	.042	.040	.038	.035	.032	.028	.025	.021	.013	.005	.000	-.004	-.009	-.011
.60	.035	.034	.032	.030	.027	.024	.020	.017	.013	.006	-.000	-.006	-.012	-.017	-.019
.62	.027	.026	.024	.021	.018	.015	.012	.009	.004	-.001	-.007	-.013	-.020	-.024	-.026
.64	.017	.016	.014	.012	.009	.006	.003	.000	-.003	-.010	-.017	-.023	-.028	-.032	-.033
.66	.006	.005	.004	.002	.000	-.002	-.005	-.008	-.012	-.020	-.026	-.032	-.037	-.041	-.041
.68	-.003	-.004	-.005	-.007	-.010	-.013	-.015	-.017	-.021	-.028	-.034	-.041	-.046	-.049	-.049

Table 1.—Coordinates of lower nappe for selected values of  $h_s/P$  for sharp-crested weir—Continued

$z/h_s$	0.01	0.10	0.20	0.40	0.60	0.80	1.00	1.20	1.50	2.00	2.50	3.00	3.50	4.00	4.50
	$y/h_s$														
Upstream face sloped 1:3															
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.01	.017	.017	.016	.016	.014	.013	.012	.011	.010	.010	.009	.008	.008	.007	.006
.02	.026	.026	.025	.025	.025	.024	.023	.022	.021	.019	.019	.016	.015	.013	.011
.03	.036	.036	.035	.033	.033	.032	.031	.030	.029	.026	.024	.023	.021	.018	.016
.04	.043	.043	.042	.041	.040	.038	.039	.037	.036	.033	.031	.029	.027	.023	.021
.05	.049	.048	.048	.047	.046	.045	.044	.043	.042	.039	.037	.035	.032	.028	.023
.06	.054	.054	.053	.052	.051	.050	.049	.048	.046	.044	.042	.040	.036	.031	.026
.07	.059	.058	.058	.056	.055	.054	.053	.052	.052	.049	.046	.043	.039	.035	.029
.08	.063	.062	.062	.060	.059	.058	.057	.056	.056	.052	.049	.047	.043	.038	.032
.09	.068	.067	.066	.064	.064	.063	.061	.060	.059	.055	.053	.049	.046	.041	.034
.10	.069	.068	.068	.067	.067	.066	.065	.063	.061	.058	.056	.052	.048	.044	.035
.11	.072	.071	.071	.069	.069	.068	.067	.066	.064	.062	.058	.055	.050	.046	.037
.12	.074	.073	.073	.072	.070	.069	.069	.067	.064	.060	.056	.052	.047	.043	.039
.13	.076	.075	.075	.074	.072	.072	.071	.070	.068	.065	.062	.058	.054	.048	.040
.14	.078	.077	.077	.076	.074	.074	.073	.072	.069	.066	.063	.059	.055	.049	.041
.15	.080	.080	.079	.078	.077	.075	.075	.073	.072	.068	.064	.060	.056	.051	.043
.16	.082	.081	.081	.079	.078	.076	.076	.074	.073	.069	.065	.061	.056	.052	.043
.17	.083	.082	.082	.080	.079	.077	.077	.076	.074	.070	.066	.062	.057	.053	.043
.18	.084	.083	.083	.081	.080	.078	.078	.076	.075	.071	.067	.063	.058	.053	.043
.19	.085	.084	.084	.081	.081	.079	.079	.077	.075	.071	.068	.064	.058	.053	.043
.20	.085	.084	.084	.082	.080	.079	.079	.078	.076	.072	.068	.064	.059	.053	.042
.21	.087	.086	.085	.082	.081	.079	.079	.078	.076	.072	.068	.064	.059	.053	.042
.22	.086	.085	.085	.083	.082	.080	.080	.078	.076	.072	.068	.064	.059	.053	.041
.23	.085	.084	.084	.082	.082	.080	.080	.078	.076	.072	.068	.064	.059	.053	.040
.24	.085	.084	.084	.082	.081	.080	.080	.078	.076	.072	.068	.064	.059	.053	.039
.25	.085	.084	.084	.082	.081	.079	.079	.078	.075	.071	.067	.064	.059	.052	.038
.26	.085	.084	.084	.082	.081	.079	.079	.077	.074	.070	.066	.062	.058	.051	.037
.27	.084	.083	.083	.081	.080	.079	.078	.077	.074	.070	.065	.062	.057	.050	.036
.28	.083	.082	.082	.080	.079	.077	.077	.076	.073	.069	.065	.062	.056	.049	.035
.29	.083	.082	.082	.079	.078	.077	.076	.075	.073	.068	.065	.061	.056	.048	.034
.30	.082	.081	.081	.078	.077	.076	.075	.074	.072	.068	.065	.061	.056	.047	.032
.31	.080	.079	.079	.077	.076	.075	.074	.073	.070	.068	.063	.059	.054	.046	.031
.32	.079	.078	.078	.076	.075	.074	.073	.072	.069	.066	.062	.057	.052	.045	.029
.33	.078	.077	.077	.075	.073	.073	.072	.070	.068	.064	.060	.056	.050	.042	.027
.34	.076	.075	.075	.073	.072	.071	.070	.069	.067	.063	.059	.054	.049	.040	.025
.35	.074	.073	.073	.072	.070	.070	.069	.068	.065	.062	.057	.053	.048	.039	.023
.36	.073	.072	.072	.070	.069	.068	.067	.066	.063	.060	.056	.052	.046	.038	.021
.37	.072	.071	.071	.068	.067	.066	.066	.064	.061	.058	.054	.050	.044	.036	.019
.38	.070	.069	.069	.066	.065	.065	.064	.062	.059	.056	.052	.048	.042	.034	.017
.39	.068	.067	.067	.064	.063	.063	.062	.060	.057	.054	.050	.046	.040	.031	.015
.40	.066	.065	.065	.062	.061	.061	.060	.058	.055	.052	.048	.045	.039	.030	.012
.42	.061	.060	.060	.058	.057	.057	.055	.053	.051	.047	.044	.040	.034	.025	.007
.44	.056	.055	.055	.053	.052	.052	.050	.049	.046	.042	.039	.036	.030	.020	.001
.46	.051	.050	.050	.048	.047	.046	.045	.044	.040	.036	.033	.031	.025	.016	—
.48	.045	.044	.044	.042	.040	.040	.039	.038	.035	.030	.028	.026	.019	.010	—
.50	.040	.039	.038	.036	.034	.034	.033	.032	.029	.024	.021	.020	.013	.006	—
.52	.032	.031	.031	.029	.028	.027	.027	.025	.022	.018	.015	.013	.006	.000	—
.54	.025	.024	.024	.022	.021	.020	.019	.018	.015	.010	.008	.007	.002	—	—
.56	.018	.017	.017	.015	.014	.013	.012	.011	.008	.003	.000	—	—	—	—
.58	.009	.008	.008	.007	.007	.006	.005	.003	.000	—	—	—	—	—	—
.60	.000	.000	.000	—	.000	—	.003	—	.003	—	—	—	—	—	—
.62	—	.007	—	.008	—	.011	—	.010	—	.017	—	.020	—	.022	—
Upstream face sloped 2:3 [Asterisk (*) indicates no data available]															
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	(*)	(*)
.01	.009	.009	.009	.008	.008	.007	.006	.006	.006	.005	.004	.004	.004	(*)	(*)
.02	.016	.016	.016	.014	.014	.014	.013	.012	.010	.009	.008	.008	.008	(*)	(*)
.03	.024	.023	.023	.021	.021	.020	.020	.019	.018	.016	.013	.012	.012	(*)	(*)
.04	.030	.029	.029	.027	.027	.026	.025	.024	.023	.021	.019	.017	.016	(*)	(*)
.05	.035	.034	.034	.032	.032	.031	.030	.029	.028	.026	.024	.021	.020	(*)	(*)
.06	.040	.039	.039	.037	.036	.035	.035	.034	.032	.030	.027	.025	.023	(*)	(*)
.07	.044	.043	.043	.042	.040	.039	.039	.037	.036	.034	.030	.028	.027	(*)	(*)
.08	.048	.048	.047	.046	.044	.043	.042	.041	.040	.037	.034	.032	.030	(*)	(*)
.09	.051	.050	.050	.049	.048	.047	.046	.045	.043	.040	.037	.035	.033	(*)	(*)
.10	.054	.053	.053	.051	.051	.050	.049	.048	.047	.046	.042	.040	.037	(*)	(*)
.11	.056	.056	.055	.054	.053	.052	.051	.049	.048	.045	.042	.040	.036	(*)	(*)
.12	.058	.057	.057	.055	.055	.054	.053	.051	.050	.048	.045	.042	.041	(*)	(*)
.13	.060	.059	.059	.057	.056	.056	.054	.053	.049	.048	.045	.042	.041	(*)	(*)
.14	.061	.060	.060	.059	.058	.057	.056	.055	.053	.051	.048	.045	.044	(*)	(*)
.15	.062	.061	.061	.060	.059	.059	.057	.056	.055	.052	.049	.047	.046	(*)	(*)
.16	.063	.062	.062	.061	.060	.059	.058	.057	.056	.053	.051	.048	.047	(*)	(*)
.17	.064	.063	.063	.062	.061	.061	.060	.059	.058	.055	.052	.049	.048	(*)	(*)
.18	.064	.063	.063	.062	.061	.061	.060	.059	.058	.055	.052	.049	.048	(*)	(*)
.19	.065	.064	.064	.063	.062	.061	.060	.059	.057	.055	.053	.050	.050	(*)	(*)
.20	.065	.064	.064	.063	.063	.059	.062	.060	.058	.055	.054	.051	.051	(*)	(*)
.21	.064	.064	.064	.063	.063	.061	.061	.060	.058	.055	.054	.051	.051	(*)	(*)
.22	.064	.064	.063	.063	.063	.061	.060	.059	.057	.055	.053	.051	.051	(*)	(*)
.23	.064	.063	.063	.062	.062	.060	.060	.059	.057	.055	.053	.050	.051	(*)	(*)
.24	.063	.063	.062	.062	.062	.060	.059	.058	.056	.054	.052	.050	.050	(*)	(*)
.25	.062	.061	.061	.061	.061	.059	.058	.057	.055	.053	.052	.050	.050	(*)	(*)
.26	.061	.061	.060	.060	.059	.058	.057	.056	.054	.053	.052	.049	.049	(*)	(*)
.27	.060	.060	.059	.059	.058	.056	.055	.055	.053	.052	.050	.047	.048	(*)	(*)
.28	.059	.058	.058	.058	.057	.055	.055	.054	.052	.051	.049	.047	.048	(*)	(*)
.29	.057	.057	.056	.056	.055	.054	.053	.052	.051	.048	.048	.046	.046	(*)	(*)
.30	.056	.056	.055	.054	.053	.052	.051	.051	.049	.048	.047	.045	.045	(*)	(*)
.31	.054	.054	.053	.053	.052	.050	.050	.048	.046	.045	.043	.044	.044	(*)	(*)
.32	.052	.052	.051	.051	.050	.049	.048	.048	.047	.044	.041	.042	.042	(*)	(*)
.33	.050	.050	.049	.049	.048	.048	.048	.046	.045	.042	.042	.040	.040	(*)	(*)

Table 1.—Coordinates of lower nappe for selected values of  $h_s/P$  for sharp-crested weir—Continued

$x/h_s$	0.01	0.10	0.20	0.40	0.60	0.80	1.00	1.20	1.50	2.00	2.50	3.00	3.50	4.00	4.50
	$y/h_s$														
<b>Upstream face sloped 2:3—Continued</b>															
.34	.047	.047	.046	.046	.045	.045	.045	.044	.043	.040	.040	.038	.038	(*)	(*)
.35	.045	.045	.044	.044	.043	.042	.042	.041	.038	.037	.037	.036	.036	(*)	(*)
.36	.043	.043	.042	.042	.041	.041	.040	.039	.036	.035	.035	.033	.033	(*)	(*)
.37	.041	.041	.040	.040	.039	.039	.038	.037	.034	.033	.031	.031	.032	(*)	(*)
.38	.038	.038	.037	.037	.036	.036	.035	.035	.032	.031	.029	.029	.029	(*)	(*)
.39	.036	.036	.035	.035	.034	.033	.033	.032	.030	.029	.027	.027	.027	(*)	(*)
.40	.034	.034	.033	.033	.032	.031	.031	.030	.028	.027	.025	.025	.025	(*)	(*)
.41	.031	.031	.030	.030	.029	.028	.028	.027	.025	.024	.022	.022	.023	(*)	(*)
.42	.028	.028	.027	.027	.026	.026	.026	.025	.023	.022	.021	.019	.018	(*)	(*)
.43	.024	.024	.024	.025	.025	.023	.024	.023	.022	.021	.019	.018	.017	(*)	(*)
.44	.021	.021	.021	.022	.022	.020	.020	.020	.019	.018	.016	.015	.014	(*)	(*)
.45	.018	.018	.018	.019	.019	.017	.017	.017	.016	.015	.013	.012	.011	(*)	(*)
.46	.015	.015	.015	.016	.016	.014	.014	.014	.013	.012	.011	.009	.008	(*)	(*)
.47	.012	.012	.012	.013	.013	.011	.012	.011	.010	.009	.008	.007	.005	(*)	(*)
.48	.009	.009	.009	.010	.009	.008	.008	.008	.007	.007	.005	.003	.003	(*)	(*)
.49	.005	.005	.005	.006	.006	.004	.004	.004	.003	.003	.002	.001	.000	(*)	(*)
.50	.003	.003	.002	.002	.002	.001	.001	.001	.000	.000	.000	.000	.000	(*)	(*)
<b>Upstream face sloped 3:3</b>															
[Asterisk (*) indicates no data available]															
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	(*)	(*)
.01	.004	.004	.004	.004	.004	.004	.004	.004	.004	.003	.003	.003	.003	(*)	(*)
.02	.009	.009	.008	.008	.008	.008	.007	.007	.007	.006	.006	.006	.006	(*)	(*)
.03	.013	.013	.013	.012	.012	.012	.011	.011	.011	.010	.010	.009	.009	(*)	(*)
.04	.016	.016	.016	.015	.015	.015	.015	.015	.014	.013	.012	.012	.012	(*)	(*)
.05	.020	.020	.020	.018	.018	.018	.018	.018	.017	.016	.015	.015	.015	(*)	(*)
.06	.023	.023	.023	.022	.022	.022	.021	.021	.021	.020	.018	.018	.018	(*)	(*)
.07	.026	.026	.026	.025	.025	.025	.025	.024	.024	.023	.022	.022	.022	(*)	(*)
.08	.029	.029	.029	.028	.028	.028	.027	.027	.027	.026	.024	.024	.024	(*)	(*)
.09	.032	.032	.032	.031	.031	.030	.030	.030	.029	.029	.028	.028	.028	(*)	(*)
.10	.036	.035	.035	.034	.034	.033	.032	.032	.031	.031	.030	.028	.028	(*)	(*)
.11	.038	.038	.037	.036	.036	.035	.035	.034	.033	.032	.030	.030	.030	(*)	(*)
.12	.040	.040	.039	.039	.038	.037	.037	.036	.035	.034	.032	.032	.032	(*)	(*)
.13	.041	.041	.040	.040	.040	.039	.039	.038	.037	.036	.034	.034	.034	(*)	(*)
.14	.042	.042	.041	.041	.041	.040	.040	.039	.038	.037	.035	.035	.035	(*)	(*)
.15	.043	.043	.042	.042	.042	.041	.041	.040	.038	.037	.036	.036	.036	(*)	(*)
.16	.044	.044	.043	.043	.043	.042	.041	.041	.040	.039	.037	.037	.037	(*)	(*)
.17	.044	.044	.044	.043	.043	.043	.042	.042	.041	.040	.038	.038	.038	(*)	(*)
.18	.044	.044	.044	.043	.043	.043	.043	.042	.041	.040	.038	.038	.038	(*)	(*)
.19	.044	.044	.044	.043	.043	.043	.043	.043	.042	.041	.039	.039	.039	(*)	(*)
.20	.044	.044	.044	.043	.043	.043	.043	.043	.042	.042	.039	.039	.039	(*)	(*)
.21	.044	.044	.044	.043	.043	.043	.043	.043	.042	.042	.039	.039	.039	(*)	(*)
.22	.044	.044	.044	.043	.043	.043	.043	.043	.042	.042	.039	.039	.039	(*)	(*)
.23	.043	.043	.043	.042	.042	.042	.042	.042	.041	.041	.039	.038	.038	(*)	(*)
.24	.042	.042	.042	.041	.042	.041	.041	.041	.040	.040	.039	.038	.038	(*)	(*)
.25	.041	.041	.041	.040	.041	.041	.041	.041	.040	.040	.038	.037	.037	(*)	(*)
.26	.040	.040	.040	.039	.040	.040	.040	.039	.039	.040	.039	.037	.036	(*)	(*)
.27	.038	.038	.038	.038	.039	.038	.038	.039	.038	.038	.039	.036	.035	(*)	(*)
.28	.037	.037	.037	.037	.037	.037	.037	.037	.037	.037	.037	.035	.034	(*)	(*)
.29	.036	.036	.036	.036	.036	.036	.036	.036	.035	.035	.034	.034	.034	(*)	(*)
.30	.034	.034	.034	.034	.034	.034	.034	.034	.033	.034	.034	.032	.032	(*)	(*)
.31	.032	.032	.032	.032	.033	.032	.032	.032	.032	.032	.032	.031	.031	(*)	(*)
.32	.030	.030	.030	.030	.031	.031	.031	.031	.031	.030	.030	.029	.029	(*)	(*)
.33	.028	.028	.028	.028	.029	.029	.029	.029	.029	.029	.029	.027	.027	(*)	(*)
.34	.026	.026	.026	.026	.026	.026	.027	.027	.027	.027	.028	.025	.026	(*)	(*)
.35	.023	.023	.023	.024	.024	.024	.024	.024	.025	.025	.026	.024	.024	(*)	(*)
.36	.020	.020	.020	.020	.022	.022	.021	.022	.022	.022	.022	.022	.022	(*)	(*)
.37	.018	.018	.018	.018	.019	.020	.020	.020	.020	.020	.020	.020	.019	(*)	(*)
.38	.015	.015	.016	.016	.017	.017	.018	.018	.017	.018	.018	.017	.017	(*)	(*)
.39	.011	.011	.012	.013	.014	.014	.015	.015	.016	.015	.016	.015	.014	(*)	(*)
.40	.009	.009	.010	.010	.011	.011	.012	.012	.013	.013	.014	.013	.011	(*)	(*)
.41	.006	.006	.006	.007	.008	.008	.009	.009	.010	.010	.011	.011	.009	(*)	(*)
.42	.002	.003	.003	.003	.005	.006	.006	.006	.007	.007	.009	.008	.007	(*)	(*)
.43	-.001	-.001	.000	.000	.002	.003	.004	.004	.004	.005	.006	.006	.005	(*)	(*)
.44	-.002	-.002	-.002	-.001	-.001	-.000	.001	.001	.002	.002	.003	.004	.002	(*)	(*)
.45	-.007	-.006	-.006	-.004	-.003	-.003	-.001	-.002	-.001	-.000	.001	.001	.000	(*)	(*)
.46	-.010	-.010	-.009	-.008	-.007	-.006	-.005	-.005	-.004	-.003	-.002	-.001	-.003	(*)	(*)
.47	-.014	-.014	-.013	-.012	-.011	-.010	-.008	-.009	-.009	-.006	-.005	-.005	-.005	(*)	(*)
.48	-.018	-.017	-.017	-.015	-.014	-.014	-.012	-.012	-.011	-.009	-.008	-.007	-.008	(*)	(*)
.49	-.022	-.021	-.021	-.019	-.018	-.017	-.016	-.015	-.014	-.011	-.011	-.010	-.011	(*)	(*)
.50	-.026	-.025	-.025	-.023	-.022	-.021	-.019	-.019	-.018	-.016	-.014	-.013	-.015	(*)	(*)

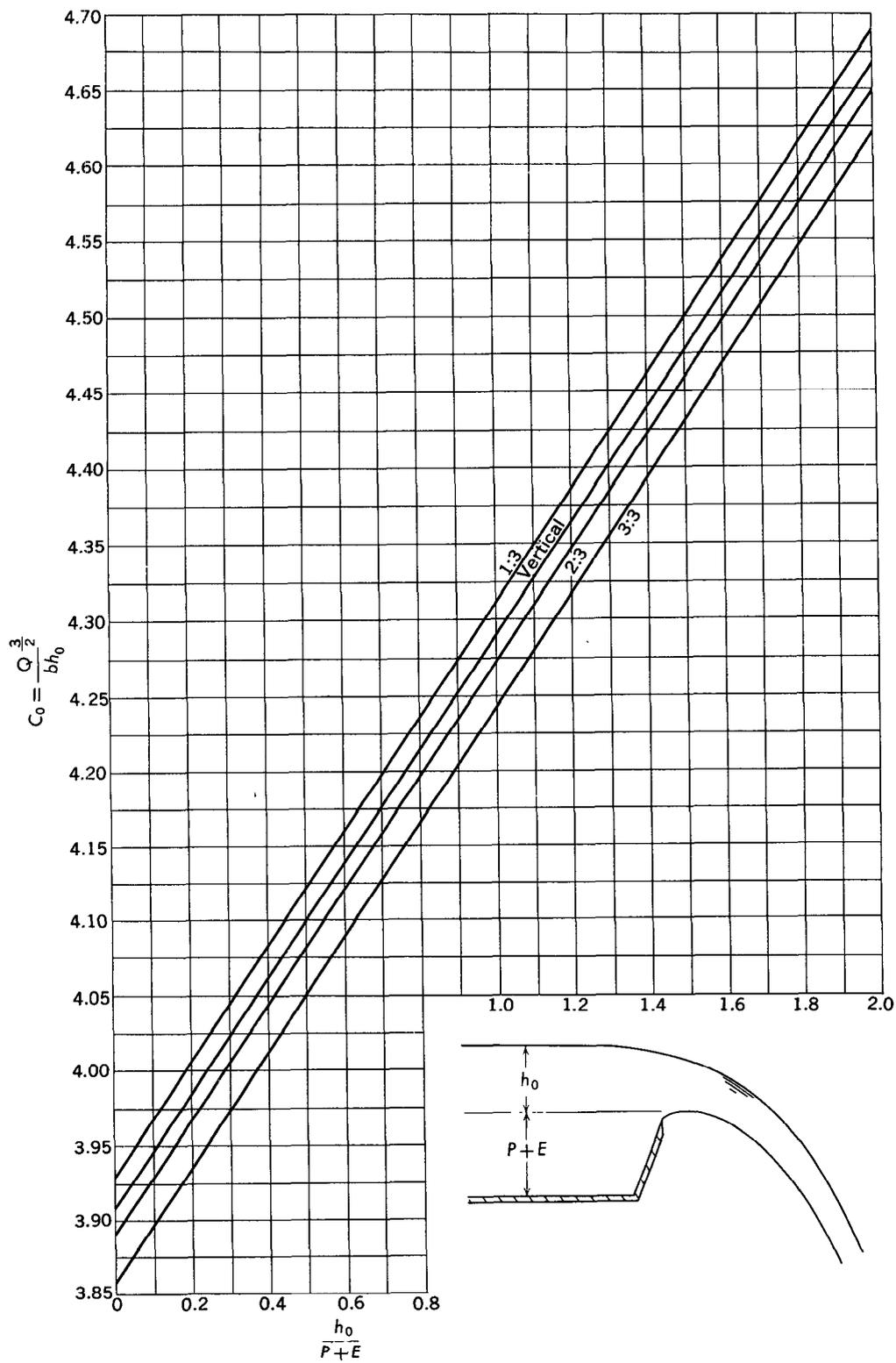


Figure 15.—Discharge coefficients corresponding to the design head,  $h_0$ , for full width ogee dams for various values of  $h_0/(P+E)$ .

## Head-discharge coefficient relationship

The discharge coefficient corresponding to any head on the dam may be determined by the following procedure:

1. Determine the coefficient,  $C_0$ , corresponding to the design head  $h_0$  from figure 15.
2. Determine the ratio of actual static head,  $h$ , to the design head,  $h_0$ .
3. Determine the ratio  $C/C_0$  from figure 16 and compute the value of  $C$ . This  $C$  is applicable to full width ( $b/B=1.0$ ) dams. For all  $b/B$  ratios less than 1.0,  $C$  must be corrected by  $k_c$  from figure 3.

## Example.

The nappe-fitting method was used to compute the peak discharge over the crest shown on figure 17. The profile is for an overfall dam, 33.4 feet high. The upstream face is inclined slightly from the vertical, with a slope of 1 in 12, but it was assumed in the computation to be vertical. The crest length is 400 feet with no side contraction.

The nappe profiles from table 1 corresponding to assumed values of  $h_s$  of 14.0, 16.0, and 18.0 were plotted on figure 18 and compared with the crest profile of the dam. A value of 16.3 for  $h_s$  was selected on the basis of this comparison.

The value of  $h_s/P$  was computed as 0.49; the corresponding value of  $E$  from table 1 is  $0.108 \times 16.3 = 1.8$ . The value of  $h_0$  is then

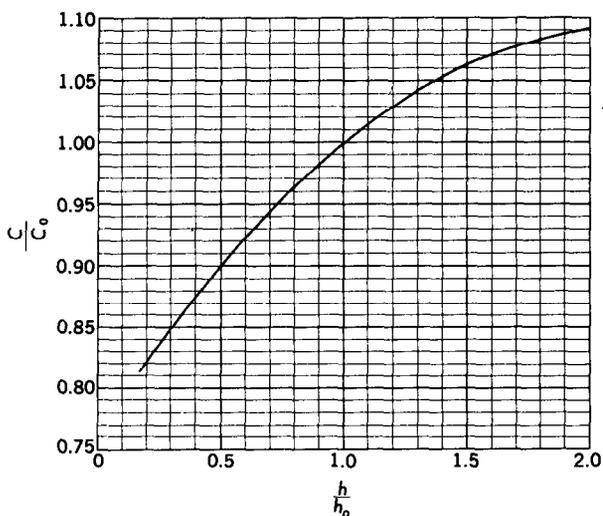


Figure 16.—Discharge coefficients for heads other than the design head, nappe-fitting weirs.

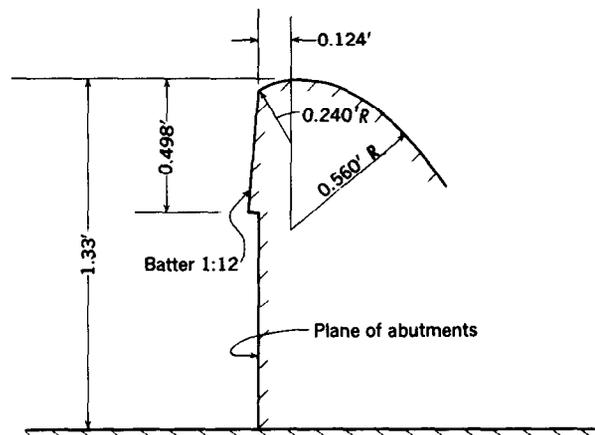


Figure 17.—Profile of North Highlands spillway model.

$16.3 - 1.8 = 14.5$ . The value of  $h_0/(P+E)$  is 0.412, and  $C_0$  from figure 15 is 4.07.

The actual head on the dam from high-water marks was 18.00 feet, and the ratio  $h/h_0$  is  $18.00/14.50$  or 1.24.

The value of  $C/C_0$  from figure 16 is 1.03, and  $C$  is 4.20.

The peak discharge by equation 1 is 128,000 cfs.

## Index-measurement method

The index-measurement method depends on a measurement of discharge to define the discharge coefficient  $C_i$  for a corresponding total head,  $H_i$ . The coefficient for other heads may then be determined from the relationship shown in figure 19 between  $C/C_i$  and  $H/H_i$ . This method which was developed by Kindsvater<sup>2</sup> is applicable to any round-crested dam. Its advantages are that uncertainty in the value of the reference head and the coefficient for an irregular crest is avoided; the nappe-fitting procedure is eliminated, and the crest profile need not be known, although it must be ascertained that it is round crested. The major disadvantage is that a direct measurement of discharge at a head of 1 foot or more is required to define the value of  $C_i$ .

Figure 19 is applicable only to full width weirs with free flow. The value of  $C_i$  should represent or be adjusted to this condition.

<sup>2</sup> C. E. Kindsvater, 1955, A note on discharge for round-crested weirs: written communication.

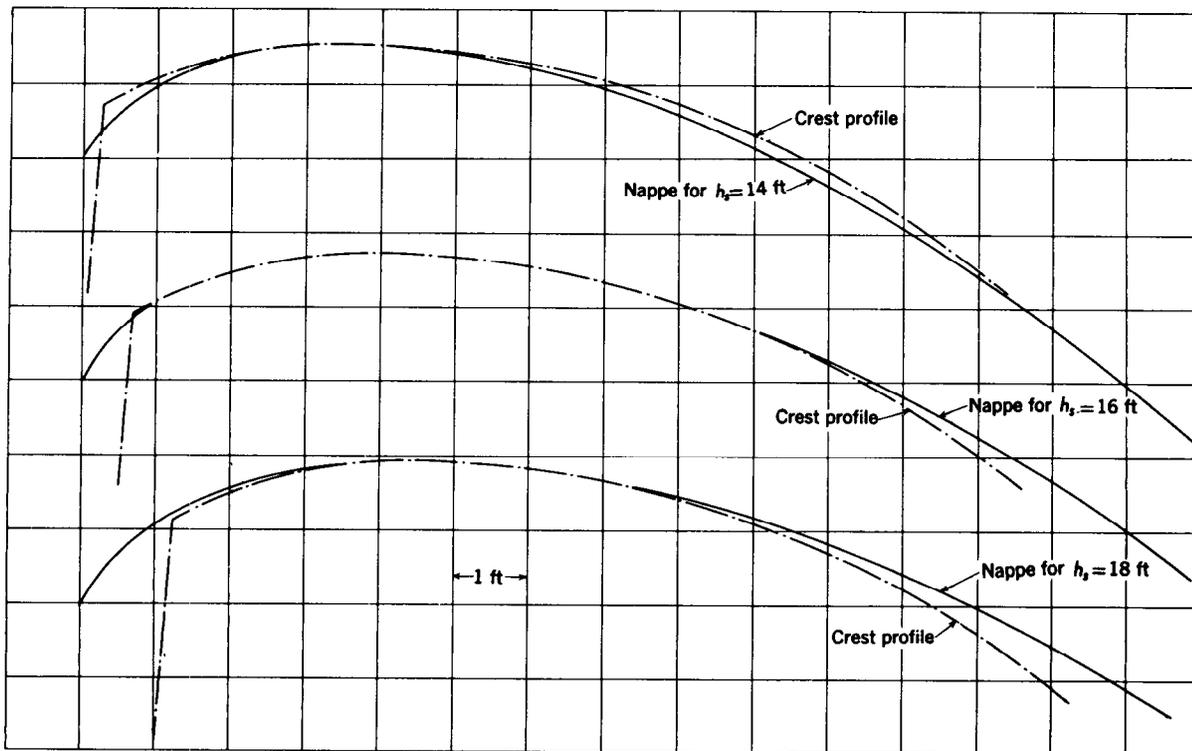


Figure 18.—Determination of nappe-fitting head, North Highlands Dam.

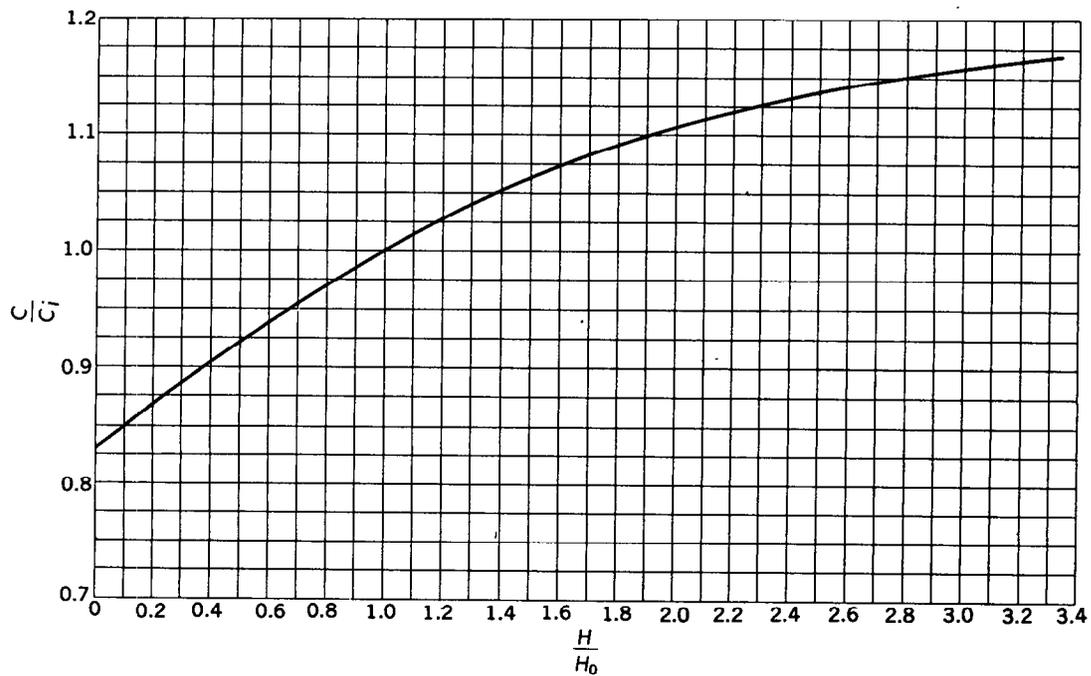


Figure 19.—Discharge coefficients for head other than the index head, round-crested weirs.

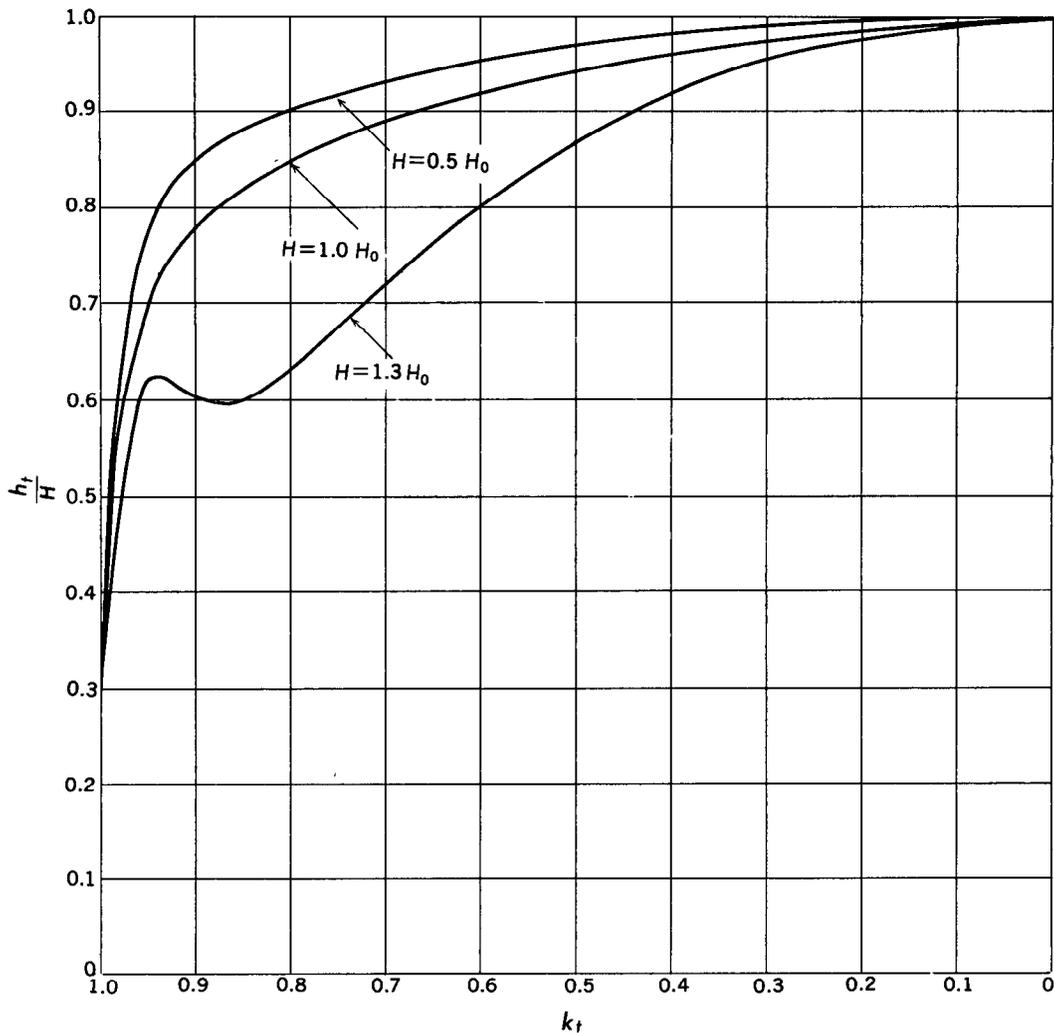


Figure 20.—Definition of adjustment factor,  $k_t$ , for submerged ogee dams.

### Effect of abutments and piers

The effect of side contraction on the discharge coefficient for round-crested weirs has not been defined by experiment. In the absence of a better solution, the value of  $C$  determined by the procedures given in the preceding sections should be multiplied by the value of  $k_c$  from figure 3 if the side abutments have square corners. It is further assumed that the value of  $k_c$  is 0 if the abutment corners are rounded, and  $r/b$  is equal to or greater than 0.12. At other values of  $r/b$  the value of  $k_c$  may be obtained by interpolating between these two values.

### Submerged round-crested weirs

The degree of submergence of a round-crested weir is defined by the ratio  $h_t/H$  as illustrated on figure 8. The effect of submergence on the discharge coefficient for design-head dams is shown on figure 20. The factor  $k_t$  is multiplied by the coefficient for free-flow conditions to account for the effect of submergence. The total head,  $H_0$ , used in this relation corresponds to the design head  $h_0$  and is equal to  $h_0 + V_0^2/2g$  for the reference weir. The downstream head,  $h_t$ , should be measured downstream from the plunging jet or hydraulic jump or energy recovery zone below the spillway.

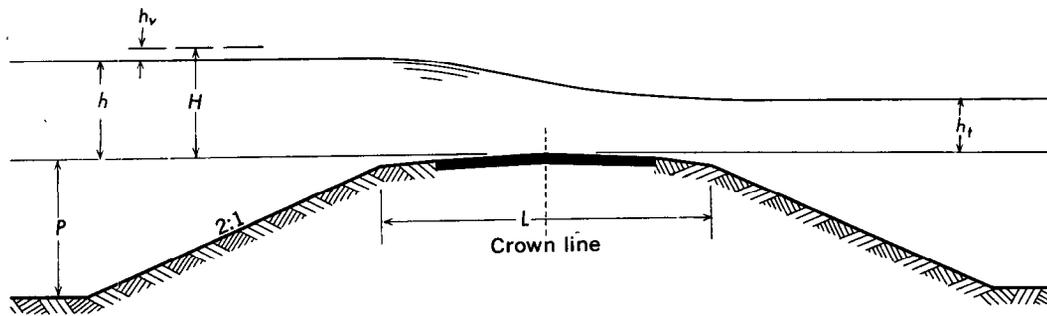


Figure 21.—Definition sketch of flow over a highway embankment.

If the degree of submergence is greater than 0.6, the computed discharge may not be reliable and other methods of determining the peak discharge should be investigated.

## Highway Embankments

It is sometimes necessary to compute flow over highway embankments in combination with flow through bridge openings. The general procedure for dividing the flow and establishing the approach section and water surface elevations for this type of indirect measurement was explained in chapter A4 by Matthai (1967). The discharge equation and coefficients for flow over a highway embankment are given in this section.

### Definitions

The geometry and flow pattern for a highway embankment are illustrated on figure 21. Under free-flow conditions critical depths occur near the crown line. The head is referred to the elevation of the crown, and the length,  $L$ , in direction of flow is the distance between the top points of the upstream and downstream embankment faces. The height of the embankment has no influence on the discharge coefficient.

### Basic equation

The discharge equation for flow over roadways is referred to the total head,  $H$ , and is

$$Q = CbH^{3/2}, \quad (3)$$

where

$Q$  = discharge,

$C$  = coefficient of discharge,

$b$  = length of the flow section along the road normal to the direction of flow, and

$H$  = total head =  $h + V_1^2/2g$ .

### Length of flow section

Because of shallow depths over the road and very flat longitudinal slope (normal to flow) of the roadway, it is often difficult to determine the length of the flow section,  $b$ , to be used in equation 3. It is thus convenient to assume that the elevation of the water surface at the crown line of the roadway (where  $b$  is measured) is equal to five-sixths of the maximum value of  $H$  for the section.

### Discharge coefficients

Extensive studies of flow over roadways were reported by Yarnell and Nagler (1930) and by Kindsvater (1964). These studies indicate that the discharge coefficient for free flow is a function of  $h/L$  when  $h/L$  is greater than 0.15. Below this value the coefficient is a function of the head,  $h$ , and the roughness of the roadway.

The discharge coefficient is defined as a function of  $h/L$  on figure 22 for the condition  $h/L > 0.15$ . The upper curve should be used for paved highways and the lower curve for graveled highways.

The discharge coefficient is defined as a function of head for the condition  $h/L < 0.15$

on figure 23. The upper curve should be used for paved highways and the lower curve for gravel highways.

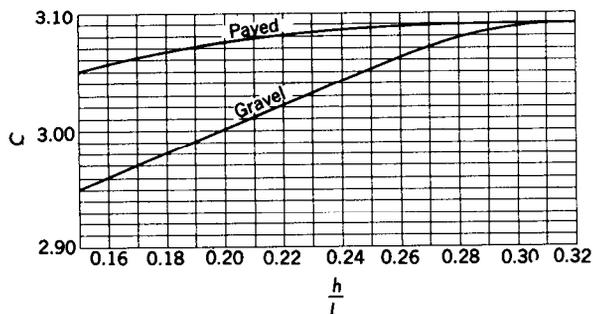


Figure 22.—Discharge coefficients for highway embankments for  $h/L$  ratios  $> 0.15$ .

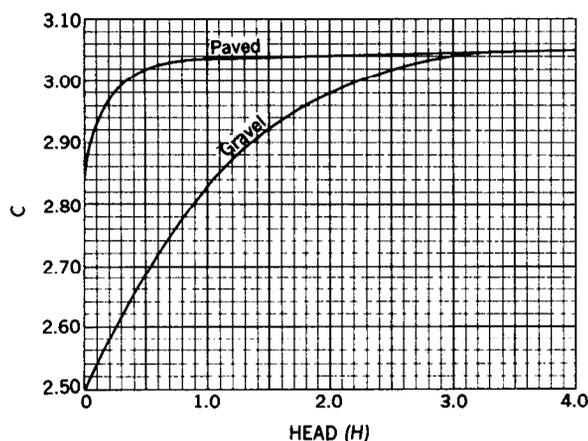


Figure 23.—Discharge coefficients for highway embankments for  $h/L$  ratios  $< 0.15$ .

### Submerged highway embankments

The degree of submergence of a highway embankment is defined by the ratio  $h_t/H$  as illustrated on figure 21. The effect of submergence on the discharge coefficient is expressed by the factor  $k_t$ , and the relation of  $k_t$  to the degree of submergence for paved and gravel surfaces is shown on figure 24. The factor  $k_t$  is multiplied by the discharge coefficient for free-flow conditions to obtain the discharge coefficient for submerged conditions.

If the degree of submergence is greater than 0.9, the computed discharge may not be reliable. However, in some indirect measurements the portion of the total flow which passes over the road as compared to that which went through the bridge may be small, and thus a greater error can be tolerated in this computation.

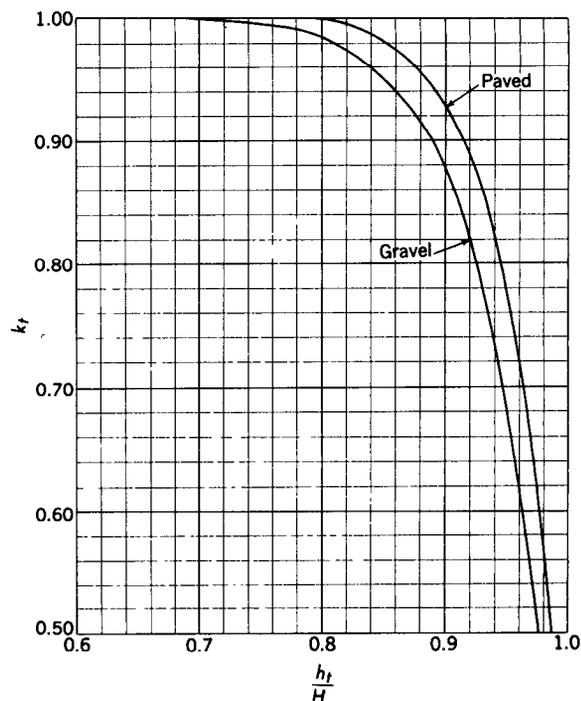


Figure 24.—Definition of adjustment factor,  $k_t$ , for submerged highway embankments.

### Weirs of Unusual Shape

Weirs of unusual shape will be found in the field that cannot be classified as sharp-, broad-, or round-crested weirs. Information obtained from tests on weirs of similar shape may be used to determine the discharge coefficient. The best source of information on weirs of unusual shape is Water-Supply Paper 200. A summary of the information from this source is given in this report. Important criteria to be used in selecting a comparative weir are similar shapes and similar ratios of head to weir height or length.

The coefficients of discharge for the various weirs in this section were computed for free-flow conditions from the equation previously given for broad-crested weirs:

$$Q = CbH^{3/2} \quad (3)$$

Weir form and pertinent measurements are shown on figure 25. Data obtained from laboratory tests on these weirs are given in tables 2-4.

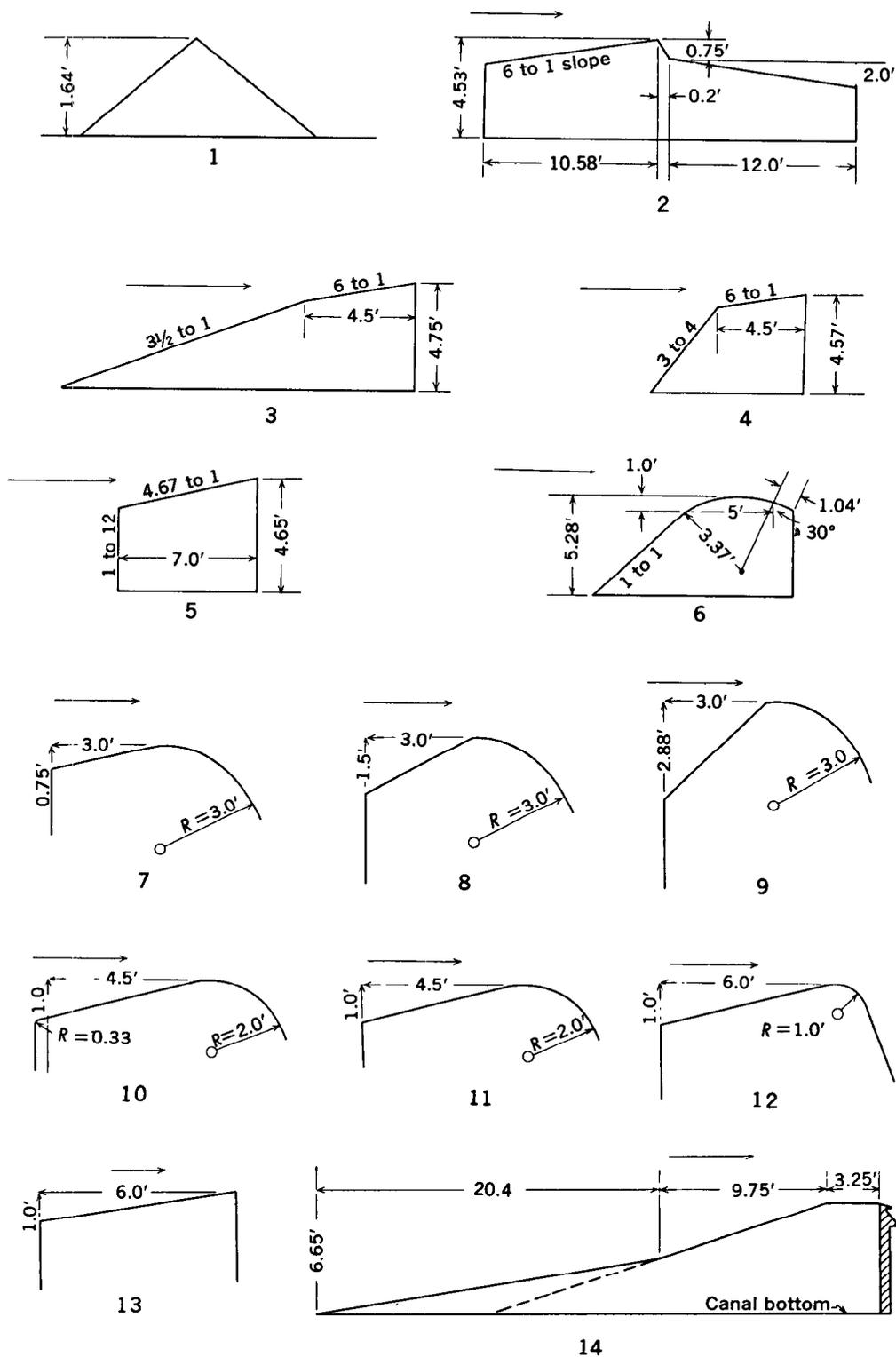


Figure 25.—Odd-shaped weir forms for which the value of  $C$  has been determined by laboratory experiment. See tables 2-4.

Table 2.—Values of C for weir form 1 on figure 25

Slope of upstream face	Slope of downstream face	Head in feet, <i>H</i>											
		0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.5	
Horizontal to vertical	Horizontal to vertical												
1:1	1:1		4.26	4.20	4.14	4.11	4.11	4.11	4.10	4.08	3.93	3.75	
1:1	2:1	3.82	3.80	3.77	3.77	3.79	3.82	3.84	3.85	3.85	3.85	3.84	
1:1	3:1		3.55	3.52	3.48	3.46	3.45	3.46	3.47	3.48	3.47	3.46	
2:1	2:1	3.88	3.85	3.83	3.81	3.81	3.83	3.86	3.87	3.87	3.87	3.87	
1:2	2:1	3.74	3.71	3.68	3.69	3.72	3.73	3.73	3.74	3.74	3.73	3.71	
1:3	2:1	3.65	3.64	3.64	3.67	3.68	3.69	3.69	3.69	3.69	3.68	3.66	

Table 3.—Values of C for weir forms 2-6 on figure 25

Weir	Head in feet, <i>H</i>									
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
2	3.13	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22
3		3.41	3.35	3.30	3.33	3.37	3.38	3.38	3.38	3.38
4	3.47	3.46	3.41	3.35	3.32	3.33	3.37	3.41	3.46	
5				3.44	3.39	3.38	3.38	3.39	3.41	
6	3.28	3.29	3.32	3.39	3.46	3.51	3.59	3.62	3.65	

Table 4.—Values of C for weir forms 7-14 on figure 25

(All weirs were 11.25 ft high and either 8 or 15 ft wide. The 8-ft weirs were contracted at one end)

Weir	Length of model (feet)	Head in feet, <i>H</i>									
		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
7	7.94		3.30	3.32	3.36	3.40	3.43	3.48	3.53	3.62	3.72
7	15.97	3.32	3.44	3.46	3.42	3.41	3.46	3.50			
8	7.98		3.38	3.46	3.51	3.55	3.58	3.62	3.68	3.74	3.83
8	15.97	3.22	3.48	3.61	3.67	3.70	3.72				
9	15.97	3.15	3.45	3.64	3.75	3.82	3.87	3.88			
10	15.97	3.23	3.34	3.43	3.52	3.59	3.64				
11	15.97	3.18	3.30	3.37	3.42	3.46	3.49	3.52	3.54		
12	15.97	3.28	3.50	3.54	3.52	3.36	3.31	3.30			
13	15.97	3.53	3.54	3.55	3.50	3.35	3.27	3.25	3.25		
14	15.93	3.13	3.14	3.10	3.14	3.20	3.26	3.31	3.37		

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